Two-stage drag of electrons by phonons in semimetals with equal electron and hole densities

V. A. Kozlov and V. D. Lakhno

Moscow Physicotechnical Institute

(Submitted October 6, 1975; resubmitted January 22, 1976)
Fiz. Tverd. Tela (Leningrad) 18, 1373–1380 (May 1976)

A theory is developed of the two-stage drag of electrons by phonons in semimetals with two types of carrier in the limit of the hydrodynamic flow of thermal phonons. By analogy with the standard Herring drag thermoelectric power, the two-stage drag thermoelectric power vanishes in pure infinite samples in which the carriers are scattered mainly by phonons and the electron and hole densities are equal. Nonzero thermoelectric power due to the phonon drag can appear only if, in addition to the phonon scattering, the carriers are scattered due to a different mechanism. The sign of the thermoelectric power is determined by the type of carrier whose mean free path is shorter in the phonon scattering case. The calculated results are in good agreement with the experimental results for bismuth provided the additional scattering mechanism is chosen to be the scattering from the surface of a sample. In contrast to the thermoelectric power, the Nernst constant due to the phonon drag is nonzero even in the absence of an additional scattering mechanism and is a factor of $(\mu/T)^2$ greater than the value corresponding to one type of carrier (μ is the Fermi energy of carriers and T is the absolute temperature).

PACS numbers: 72.15.Qm, 72.10.Di

The physical model of the two-stage drag of electrons by phonons can be described as follows. Because of the laws of conservation of the energy and momentum, carriers in semiconductors and semimetals interact only with phonons whose wavelength is of the order of the carrier wavelength.[1] Under normal conditions, the wavelength of such "electron" phonons is much longer than the wavelength of thermal phonons, whose frequency is of the order of k_BT. Therefore, Herring[1] was able to conclude that the electron phonons give rise to the phonon drag thermoelectric power in semiconductors.

It was assumed in ref. 1 that the interaction of electron phonons with thermal phonons tends to bring the electron phonons in equilibrium and, therefore, the motion of intrinsic thermal phonons in an applied temperature gradient was neglected. However, it was shown in refs. 2-4 that the deviation of thermal phonons from equilibrium may be quite large in some cases and such nonequilibrium thermal phonons cannot bring the electron phonons in thermal equilibrium. Therefore, the drag takes place in two stages: thermal phonons drag long-wavelength phonons and the long-wavelength phonons drag electrons.

Such an effect manifests itself most strongly in the limit of the hydrodynamic flow of thermal phonons when the momentum dissipation of the thermal phonons is solely due to the Umklapp scattering.[5] It was shown in refs. 2-4 that this leads to an exponential increase in the thermoelectric power due to the phonon drag. The aforementioned behavior of the thermoelectric power was observed on very pure bismuth single crystals.[6]

Only the case of carriers of equal signs was considered in refs. 2-4. However, when carriers of two different signs are considered, the effect under study exhibits new features whose explanation is required in the interpretation of experimental results.

We shall study the two-stage drag of electrons by phonons in a system with two types of carrier with equal densities, which reflects better the experimental conditions of ref. 6. When the momentum of the electron–phonon system is lost due only to the Umklapp scattering of phonons, the thermoelectric power due to the phonon drag is equal to zero. A nonzero thermoelectric power can be obtained only if one type of carrier is scattered by an additional scattering mechanism.[1)]

In particular, under the experimental conditions of ref. 6, the scattering from the surface of a sample can represent such an additional mechanism. In contrast to the thermoelectric power, the Nernst effect due to the two-stage drag of electrons by phonons does not vanish when the electron and hole densities are equal to one another. In fact, the Nernst constant is a factor of $(\mu/T)^2$ greater than in the case of a system with one type of carrier since, in addition to the partial Nernst constants, it contains terms proportional to the partial thermoelectric powers of the corresponding types of carrier.[7]

Similarly to the diffusion, the partial Nernst constants taking into account the phonon drag contain a small factor (T/μ) (the diffusion term is proportional to this factor and the drag term is proportional to the square of this factor). However, in contrast to the diffusion thermoelectric power, the partial thermoelectric power due to the drag does not contain such a factor (this applies not only to the thermoelectric power and the Nernst effect due to the two-stage drag but also to the standard Herring drag). The thermoelectric power of a system with two types of carrier was previously discussed in refs. 8 and 9. In contrast to refs. 8 and 9, we shall consider the situation when the momentum of the system under study is lost due mainly to the phonon Umklapp scattering in the system of thermal phonons. We shall also show that the standard Herring thermoelectric power due to the phonon drag vanishes when the electron density is equal to the hole density and both types of carrier are scattered only by phonons. In general, conditions can be realized when the two-stage drage of electrons by phonons influences strongly the conductivity. This effect will not be considered in the present paper and will be discussed in a separate publication.

1. GENERAL RELATIONS

We shall consider an isotropic semimetal with equal electron and hole densities. Since the phonons interact simultaneously with electrons and holes, their deviation from equilibrium depends on the state of carriers. Therefore, under the drag-effect conditions, the electron and hole currents cannot be always regarded as independent and, therefore, the standard expressions for the transport coefficients are no longer applicable for a system with two types of interacting carrier.

We shall derive a general expression for the thermoelectric power and for the Nernst effect corresponding to the two-stage drag in a two-band model. We shall neglect the direct interaction between the electrons and holes since it was shown in refs. 8 and 10 that the corresponding mean free path is about ten times as long as the mean free path due to the electron–phonon scattering in the temperature range considered.

Following ref. 9, we shall write the system of transport equations for carriers and phonons in the following form, which takes into account the deviation from equilibrium of thermal phonons and the additional scattering mechanism of carriers:

$$s\nabla T\frac{\partial F_q}{\partial T}=\mathcal{J}^N\{F_q\}+\mathcal{J}^U\{F_q\}+\mathcal{J}^N\{F_q,\,G_q\}, \tag{1}$$

$$s\nabla T\frac{\partial G_q}{\partial T}=\mathcal{J}^N\{G_q,\,F_q\}+\sum_{\pm}\mathcal{J}^N\{G_q,\,f_q^{\pm}\}, \tag{2}$$

$$e^{\pm}\left\{\mathbf{E}+\frac{1}{c}[\mathbf{v}_p^{\pm},\,\mathbf{H}]\right\}\frac{\partial f_p^{\pm}}{\partial p}+\mathbf{v}_p^{\pm}\nabla T\frac{\partial f_p^{\pm}}{\partial T}=\mathcal{J}^N\{f_p^{\pm},\,G_p\}-\frac{f_p^{\pm}-f_p^{0\pm}}{\tau_d^{\pm}} \tag{3}$$

where F_q and G_q are the distribution functions of the thermal and electron phonons, $f_p^{\pm}$ is the distribution function of holes (electrons), $\tau_d^{\pm}$ is the hole (electron) relaxation time due to the scattering by the additional scatterers, and s is the velocity of sound. The superscript 0 denotes the equilibrium parts of the quasiparticle distribution functions.

We shall seek the solution of the transport equations in the following form:

$$F_q=F_q^0-(\mathbf{qu})\frac{\partial F_q^0}{\partial\omega_q} \tag{4}$$

$$G_q=G_q^0-(\mathbf{qU})\frac{\partial G_q^0}{\partial\omega_q} \tag{5}$$

$$f_q^{\pm}=f_q^{0\pm}-(\mathbf{qV}^{\pm})\frac{\partial f_q^{0\pm}}{\partial\varepsilon_q} \tag{6}$$

where U(q), U(q), and $V^{\pm}(\varepsilon_q)$ are unknown vectors. The linearized collision integrals of phonons with phonons are given by

$$\mathcal{J}^N\{F_q,\,G_q\}=\frac{2\pi}{\hbar}\beta\sum_{q_1}(\psi_{q+q_1}-\psi_q-\varphi_{q_1})G_{q_1}^0F_q^0(1+F_{q+q_1}^0)\,|\Phi|^2\,\delta(\omega_{q_1}+\omega_q-\omega_{q+q_1}) \tag{7}$$

and

$$\mathcal{J}^N\{G_q,\,F_q\}=\frac{2\pi}{\hbar}\beta\sum_{q_1}(\varphi_q-\psi_{q+q_1}+\psi_{q_1})\,G_q^0F_{q_1}^0(1+F_{q+q_1}^0)\,|\Phi|^2\,\delta(\omega_q+\omega_{q_1}-\omega_{q+q_1}), \tag{8}$$

where φ_q = qU, ψ_q = qu, $\beta = 1/kT$; Φ is the matrix element of the corresponding three-phonon scattering (the collision integrals take into account the fact that the three-phonon scattering with the participation of electron phonons in which two thermal phonons are absorbed or emitted simultaneously are not allowed). The electron–phonon collision integrals were given in ref. 11.

It follows from ref. 5 that the solution of Eq. (1) in the principal approximation with respect to the small parameter l^N/l^U (where l^N and l^U are the mean free paths corresponding to the normal and Umklapp scattering of thermal phonons) is given by an expression describing the drift of thermal phonons in the direction of the temperature gradient with a velocity $u = -\gamma\tau^U\nabla T$, where $\gamma\approx s^2/T$; τ^U is the relaxation time corresponding to the Umklapp scattering of thermal phonons. For completely degenerate carriers, we can substitute Eqs. (4)-(8) in Eqs. (1)-(3), which yields the following system of transport equations:

$$\mathbf{v}^+(1-\gamma^+)+\frac{l^+}{r^+}[\mathbf{h},\,\mathbf{v}^+]-\gamma^+\frac{l_{ph}^+}{l_{ph}^-}\mathbf{v}^-=\frac{l^+}{l_{ph}^+}\frac{s\nabla T}{4p_F^4T}\int_0^{2p_F}Lq^3dq-\frac{l^+}{l_{ph}^+}\frac{\mathbf{u}}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq+\frac{l^+}{p_F}e^+\mathbf{E}, \tag{9}$$

$$\mathbf{v}^-(1-\gamma^-)+\frac{l^-}{r^-}[\mathbf{h},\,\mathbf{v}^-]-\gamma^-\frac{l_{ph}^-}{l_{ph}^+}\mathbf{v}^+=\frac{l^-}{l_{ph}^-}\frac{s\nabla T}{4p_F^4T}\int_0^{2p_F}Lq^3dq-\frac{l^-}{l_{ph}^-}\frac{\mathbf{u}}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq+\frac{l^-}{p_F}e^-\mathbf{E}. \tag{10}$$

Here,

$$r^{\pm}=\frac{cp_F}{e^{\pm}H};\quad\gamma^{\pm}=\frac{1}{4p_F^4}\frac{l^{\pm}}{l_{ph}^{\pm}}\int_0^{2p_F}\frac{L}{L^{\pm}}q^3dq;\quad\mathbf{h}=\frac{\mathbf{H}}{H};$$

p_F is the Fermi momentum; $L^{\pm}$ is the mean free path of electron phonons due to their scattering by holes (electrons); L is the total mean free path of electron phonons due to the scattering by holes (electrons):

$$\frac{1}{L}=\frac{1}{L^+}+\frac{1}{L^-}+\frac{1}{L_{ph}};$$

L_{ph} is the mean free path of the electron phonons due to their scattering by thermal phonons; $l_{ph}^{\pm}$ is the mean free path of holes (electrons) due to their scattering by phonons; $l^{\pm}$ is the total mean free path of holes (electrons)

$$\frac{1}{l^{\pm}}=\frac{1}{l_{ph}^{\pm}}+\frac{1}{l_d^{\pm}}$$

where $l_d^{\pm}$ is the mean free path of holes (electrons) due to the scattering by additional scatterers.

2. CALCULATION OF THE THERMOELECTRIC POWER AND THE NERNST CONSTANT

We shall study the effect of the two-stage drag on the thermoelectric power and the Nernst constant in a two-component system. The solution of the system of equations (9)-(10) can be written in the form

$$\mathbf{v}^{\pm}=\frac{\Delta_1^{\pm}+\Delta_2^{\pm}}{\Delta} \tag{11}$$

where $\Delta_1^{\mp}$ describes the standard drag of electrons by phonons.

The drag of electrons by phonons was studied in ref. 9. The term $\Delta_2^{\pm}$ describes the two-stage drag of electrons by phonons. In a weak magnetic field, i.e., for $l^{\pm}/|r^{\pm}| \ll 1$, it follows from Eqs. (9)-(10) that Δ is given by

$$\begin{aligned}\Delta = &\left\{(1-\gamma^{+})^{2}\frac{r^{+}}{l^{+}} + \gamma^{-}\frac{r^{-}}{l^{-}}\gamma^{+}\right\}\left\{(1-\gamma^{-})^{2}\frac{r^{-}}{l^{-}} + \gamma^{-}\frac{r^{+}}{l^{+}}\gamma^{+}\right\} \\ &-\left\{\gamma^{-}(1-\gamma^{-})\frac{r^{-}}{l^{-}}\frac{l^{-}_{ph}}{l^{+}_{ph}} + (1-\gamma^{+})\frac{r^{+}}{l^{+}}\gamma^{-}\frac{l^{-}_{ph}}{l^{+}_{ph}}\right\} \\ &\times\left\{\gamma^{+}(1-\gamma^{+})\frac{r^{+}}{l^{+}}\frac{l^{+}_{ph}}{l^{-}_{ph}} + (1-\gamma^{-})\frac{r^{-}}{l^{-}}\gamma^{+}\frac{l^{+}_{ph}}{l^{-}_{ph}}\right\}\end{aligned} \tag{12}$$

The quantity $\Delta_2^{\pm}$ is given by

$$\begin{aligned}\Delta_2^{+} = &-\left\{(1-\gamma^{-})^{2}\frac{r^{-}}{l^{-}} + \gamma^{+}\frac{r^{+}}{l^{+}}\gamma^{-}\right\} \\ &\times\left\{(1-\gamma^{+})\frac{r^{+}}{l^{+}_{ph}}\frac{\mathbf{u}}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq - (1-\gamma^{+})\frac{r^{+}}{p_F}e^{+}\mathbf{E}\right. \\ &-\frac{r^{-}}{l^{-}_{ph}}\frac{\mathbf{u}}{4p_F^4}\gamma^{+}\frac{l^{+}_{ph}}{l^{-}_{ph}}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq + \frac{r^{-}}{p_F}e^{-}\gamma^{+}\frac{l^{+}_{ph}}{l^{-}_{ph}}\mathbf{E} \\ &\left.-[\mathbf{h},\mathbf{u}]\frac{l^{+}}{l^{+}_{ph}4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq + [\mathbf{h},\mathbf{E}]e^{+}\frac{l^{+}}{p_F}\right\} \\ &-\left\{\gamma^{+}(1-\gamma^{+})\frac{r^{+}}{l^{+}}\frac{l^{+}_{ph}}{l^{-}_{ph}} + (1-\gamma^{-})\frac{r^{-}}{l^{-}}\gamma^{+}\frac{l^{+}_{ph}}{l^{-}_{ph}}\right\} \\ \times&\left\{(1-\gamma^{-})\frac{r^{-}}{l^{-}_{ph}}\frac{\mathbf{u}}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq - \frac{r^{+}}{l^{+}_{ph}}\frac{\mathbf{u}}{4p_F^4}\gamma^{+}\frac{l^{+}_{ph}}{l^{-}_{ph}}\int_0^{2p_F}\frac{L}{L^{-}_{ph}}q^3dq\right. \\ &\left.+\frac{r^{+}}{p_F}e^{+}\gamma^{-}\frac{l^{-}_{ph}}{l^{+}_{ph}}\mathbf{E} - [\mathbf{h},\mathbf{u}]\frac{l^{-}}{l^{-}_{ph}4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq + [\mathbf{h},\mathbf{E}]e^{+}\frac{l^{+}}{p_F}\right\}\end{aligned} \tag{13}$$

For H = 0, the solution of Eq. (11) for $\mathbf{V}_2^{+}$ describing the two-stage drag of electrons by phonons has the form

$$\begin{aligned}\mathbf{V}_2^{+} = \frac{\Delta_2^{+}}{\Delta} = \frac{1}{(1-\gamma^{-}-\gamma^{+})}&\left\{\frac{l^{+}}{l^{+}_{ph}}\frac{\mathbf{u}}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq\right. \\ &\left.+\frac{\mathbf{E}}{p_F}\left[(1-\gamma^{-})l^{+}e^{+} + \frac{l^{+}l^{-}}{l^{-}_{ph}}\gamma^{+}e^{-}\right]\right\}.\end{aligned} \tag{14}$$

The solution for $\mathbf{V}_2^{-}$ can be obtained from Eqs. (12) and (13) by the substitution of (+) for (−) and of (−) for (+). For degenerate carriers, the electric current is given by

$$\mathbf{j} = e^{+}n(\mathbf{v}^{+} - \mathbf{v}^{-}). \tag{15}$$

The thermoelectric power can be obtained from the conditions

$$\mathbf{j} = 0, \quad \mathbf{H} = 0.$$

Using Eqs. (13) and (15), we obtain the following expression for the thermoelectric power due to the two-stage drag of electrons by phonons:

$$\alpha_2 = \frac{\dfrac{\gamma\tau^{U}}{4p_F^3}\displaystyle\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq\left(\frac{l^{-}}{l_d^{-}} - \frac{l^{+}}{l_d^{+}}\right)}{|e|\left[(1-\gamma^{-})l^{+} + (1-\gamma^{+})l^{-} - \dfrac{l^{+}l^{-}_{ph}}{l^{-}_{ph}}\gamma^{+} - \dfrac{l^{-}l^{+}_{ph}}{l^{+}_{ph}}\gamma^{-}\right]} \tag{16}$$

In the absence of the scattering by additional scatterers, i.e., in the limit $l_2^{\pm} \to \infty$, it follows from Eq. (16) that the thermoelectric power due to the two-stage drag α_2 vanishes. In the limit $l_d^{\pm} \to \infty$, the standard Herring thermoelectric power due to the drag α_1 also vanishes. The same result follows from the analysis of the result of refs. 8 and 9, where the mutual drag of electrons and phonons was studied on the basis of the Herring model and the indirect interaction of carriers via phonons was also considered. However, the aforementioned problem was not discussed specifically in refs. 8 and 9.

We shall first assume that phonons are scattered mainly from phonons and the effect of electrons and holes on phonons can be neglected.

Under these conditions, we find that $L \approx L_{ph}$, $\gamma^{\pm} \ll 1$, and, in the presence of an additional scattering mechanism, the thermoelectric power due to the two-stage drag is given by

$$\alpha_2 = \frac{1}{|e|}\frac{\gamma\tau^{U}p_F}{(l^{+}+l^{-})}\left[\frac{l^{-}}{l_d^{-}} - \frac{l^{+}}{l_d^{+}}\right] \tag{17}$$

Equation (17) describes the case when the electron and hole currents can be calculated independently. Equation (17) can be easily obtained from the expression for the thermoelectric power due to the two-stage drag in a system with one type of carrier derived in ref. 2, which can be generalized to the case of two types of carrier by the standard method (see, for example, ref. 7). When the additional scattering mechanism is scattering from the surface ($l_d^{-} = l_d^{+} = d$), it follows from Eq. (17) that the sign of the thermoelectric power depends strongly on the ratio of the mean free paths of electrons and holes due to the scattering by phonons. When the inequality $l^{-}_{ph} > l^{+}_{ph}$ is satisfied, the sign of the thermoelectric power corresponds to holes and, in the opposite case, to electrons. This result is in complete agreement with the experimental results in bismuth whose thermoelectric power is of the hole type and the electron mean free path due to the scattering by phonons is longer than the corresponding mean free path of holes.

We shall now consider the case when phonons are scattered mainly by electrons and holes, which corresponds to the limit $L^{+}, L^{-} \ll L_{ph}$. In this case, Eq. (16) takes the form

$$\alpha_2 = \frac{1}{4p_F^4}\int_0^{2p_F}\frac{L_{eh}}{L_{ph}}q^3dq\,\frac{\gamma\tau^{U}p_F}{|e|}\,\frac{l^{-}/l_d^{-} - l^{+}/l_d^{+}}{l^{+}l^{-}/l_d^{+} + l^{+}l^{-}/l_d^{-}}, \tag{18}$$

where $1/L_{eh} = 1/L^{+} + 1/L^{-}$.

For a strong scattering of phonons by electrons and holes, it follows from Eq. (18) that the thermoelectric power due to the two-stage drag of electrons by phonons decreases but the exponential dependence remains unchanged provided the drag effect does not influence the conductivity.

We shall now calculate the contribution of the two-stage drag of electrons by phonons to the Nernst effect. The Nernst constant is determined from the condition j=0. Using Eqs. (12) and (13), we obtain

$$N_2 = \frac{a_1b_2 - a_2b_1}{(a_1^2 + a_2^2)H}, \tag{19}$$

where

$$a_1=\frac{1}{\Delta}\left\{-\left[(1-\gamma^-)^2\frac{r^-}{l^-}+\gamma^+\frac{r^+}{l^+}\gamma^-\right]\left[-(1-\gamma^+)\frac{r^+}{p_F}e^+ +\frac{r^-}{p_F}e^-\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]\right.$$
$$-\left[\gamma^+(1-\gamma^+)\frac{r^+}{l^+}\frac{l^+_{ph}}{l^-_{ph}}+(1-\gamma^-)\frac{r^-}{l^-}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]\frac{r^+}{p_F}e^+\gamma^-\frac{l_{ph}}{l^+_{ph}}+\left[(1-\gamma^+)^2\frac{r^+}{l^+}-\gamma^-\frac{r^-}{l^-}\gamma^+\right]$$
$$\times\left[-(1-\gamma^-)\frac{r^-}{p_F}e^- +\frac{r^+}{p_F}e^+\gamma^-\frac{l_{ph}}{l^+_{ph}}\right]$$
$$\left.+\left[\gamma^-(1-\gamma^-)\frac{r^-}{l^-}\frac{l^-_{ph}}{l^+_{ph}}+(1-\gamma^+)\frac{r^+}{l^+}\gamma^-\frac{l_{ph}}{l^+_{ph}}\right]\frac{r^-}{p_F}e^-\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right\}$$

$$a_2=\frac{1}{\Delta}\left\{-\left[(1-\gamma^-)^2\frac{r^-}{l^-}+\gamma^+\frac{r^+}{l^+}\gamma^-\right]e^+\frac{l^+}{p_F}\right.$$
$$-\left[\gamma^+(1-\gamma^+)\frac{r^+}{l^+}\frac{l^+_{ph}}{l^-_{ph}}+(1-\gamma^-)\frac{r^-}{l^-}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]e^+\frac{l^+}{p_F}$$
$$+\left[(1-\gamma^+)^2\frac{r^+}{l^+}+\gamma^-\frac{r^-}{l^-}\gamma^+\right]e^-\frac{l^-}{p_F}+\left[\gamma^-(1\right.$$
$$\left.\left.-\gamma^-)\frac{r^-}{l^-}\frac{l^-_{ph}}{l^+_{ph}}+(1-\gamma^+)\frac{r^+}{l^+}\gamma^-\frac{l^-_{ph}}{l^+_{ph}}\right]e^-\frac{l^-}{p_F}\right\}$$

$$b_1=\frac{\gamma\tau^U}{4p_F^4\Delta}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq\left\{\left[(1-\gamma^-)^2\frac{r^-}{l^-}+\gamma^+\frac{r^+}{l^+}\gamma^-\right]\left[(1-\gamma^+)\frac{r^+}{l^+_{ph}}-\frac{r^-}{l^-_{ph}}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]\right.$$
$$+\left[\gamma^+(1-\gamma^+)\frac{r^+}{l^+}\frac{l^+_{ph}}{l^-_{ph}}+(1-\gamma^-)\frac{r^-}{l^-}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]\left[(1-\gamma^-)\frac{r^-}{l^-}-\frac{r^+}{l^+_{ph}}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]$$
$$-\left[(1-\gamma^+)^2\frac{r^+}{l^+}+\gamma^-\frac{r^-}{l^-}\gamma^+\right]\left[(1-\gamma^-)\frac{r^-}{l^-_{ph}}-\frac{r^+}{l^+_{ph}}\gamma^-\frac{l^-_{ph}}{l^+_{ph}}\right]-\left[\gamma^-(1-\gamma^-)\frac{r^-}{l^-}\frac{l^-_{ph}}{l^+_{ph}}\right.$$
$$\left.\left.+(1-\gamma^+)\frac{r^+}{l^+}\gamma^-\frac{l^-_{ph}}{l^+_{ph}}\right]\left[(1-\gamma^+)\frac{r^+}{l^+_{ph}}-\frac{r^-}{l^-_{ph}}\gamma^-\frac{l^-_{ph}}{l^+_{ph}}\right]\right\}$$

and

$$b_2=\frac{\gamma\tau^U}{4p_F^4\Delta}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq\left\{-\left[(1-\gamma^-)^2\frac{r^-}{l^-}+\gamma^+\frac{r^+}{l^+}\gamma^-\right]\frac{l^+}{l^+_{ph}}-\left[\gamma^+(1-\gamma^+)\frac{r^+}{l^+}\frac{l^+_{ph}}{l^-_{ph}}\right.\right.$$
$$\left.+(1-\gamma^-)\frac{r^-}{l^-}\gamma^+\frac{l^+_{ph}}{l^-_{ph}}\right]\frac{l^-}{l^-_{ph}}+\left[(1-\gamma^+)^2\frac{r^+}{l^+}+\gamma^-\frac{r^-}{l^-}\gamma^+\right]\frac{l^-}{l^-_{ph}}$$
$$\left.+\left[\gamma^-(1-\gamma^-)\frac{r^-}{l^-}\frac{l^-_{ph}}{l^+_{ph}}+(1-\gamma^+)\frac{r^+}{l^+}\gamma^-\frac{l^-_{ph}}{l^+_{ph}}\right]\frac{l^+}{l^+_{ph}}\right\}$$

We shall now consider the case when phonons are scattered mainly by phonons and the effect of electrons and holes on the phonons can be neglected. In this case, we obtain

$$a_1=\frac{e^+}{p_F}(l^+ + l^-);\quad b_1=\left(\frac{l^+}{l^+_{ph}}-\frac{l^-}{l^-_{ph}}\right)\gamma\tau^U,$$
$$a_2=-\frac{e^+}{p_F}\left(\frac{l^{+2}}{r^+}+\frac{l^{-2}}{r^-}\right);\quad b_2=-\gamma\tau^U\left(\frac{l^{+2}}{l^+_{ph}r^+}+\frac{l^{-2}}{l^-_{ph}r^-}\right)$$

It follows from Eq. (19) that

$$N_2=-\frac{\gamma}{c}\tau^U\frac{l^+l^-}{l^+_{ph}l^-_{ph}}\,\frac{l^+_{ph}-l^-_{ph}}{l^+-l^-}. \qquad (20)$$

Equation (20) is identical with the expression which can be obtained by a straightforward generalization of the expression for the Nernst constant appropriate to a system with one type of carrier to a system with two types of carrier under the conditions when the electron and hole currents can be regarded as independent. In the absence of an additional scattering mechanism, it follows from Eq. (20) that $N_2 \approx \gamma\tau^U/c$. We shall now compare this result with the quantity N_2 obtained in ref. 3 in the form $N_2 \approx (\gamma\tau^U/c)(T/\mu)^2$ appropriate to a system with one type of carrier. It follows from such a comparison that the Nernst constant corresponding to the two-stage drag is a factor of $(\mu/T)^2$ greater for a system with two types of carrier than in the case of a system with one type of carrier.

When the drag of phonons by electrons is strong, the expression for the Nernst constant due to the two-stage drag is rather complex and we shall not quote it explicitly. The expression for the coefficients b_2 and b_1 contains a factor $\frac{1}{4p_F^4}\int_0^{2p_F}\frac{L}{L_{ph}}q^3dq \ll 1$, which indicates that the two-stage drag of electrons by phonons has much smaller influence on the Nernst effect than the standard drag of electrons by phonons.

3. DISCUSSION OF RESULTS

Equations (16) and (17) take into account the additional scattering of electrons and holes. It follows from Eq. (17) that our results agree with the experimental results of ref. 6 only if $l^{\pm}_{ph} \approx l^{\pm}_d$, i.e., when the mean free paths of electrons and holes due to the scattering by phonons are of the order of the mean free paths due to the scattering of electrons and holes by the additional scattering mechanism. Under the experimental conditions,[6] where the measurements were made on very pure and perfect single crystals, the additional scattering mechanism may be scattering from the surface of the sample. At temperatures 2-4°K, the mean free paths are given[12] by $l^{\pm}_{ph} \sim 1$ mm, when the thickness of a sample is ~2-3 mm, the quantity $l^{\pm}_{ph}$ is of the order of $l^{\pm}_d$ provided $l^{\pm}_d$ is set equal to the thickness of the sampled. It was shown in ref. 4 that the effect of the surface on the scattering of electrons and holes, which is described by the term $v_z(\partial f^{\pm}/\partial z)$ on the right-hand side of the transport equation, can be taken into account if an additional scattering mechanism with a relaxation time $d/v^{\pm}_F$ is introduced. Therefore, in the case considered, the origin of the thermoelectric power due to the drag can be attributed to the scattering of electrons and holes from the surface. Moreover, it was shown in ref. 4 that the principal contribution is due to the term describing the two-stage drag [Eq. (17)].

In general, Eq. (1) for thermal phonons should also in include the collision integral of thermal phonons with electrons $\mathcal{J}^N\{F_q, G_q\}$. This is due to the fact that the loss of the momentum from the system of thermal phonons is caused not only by Umklapp scattering but also by the scattering of thermal phonons by electrons. In fact, the aforementioned effect is governed by the parameter $g = l^U/l^N_{p_{th}p_e}$, where $l^N_{p_{th}p_e}$ is the mean free path of thermal phonons due to their scattering by electrons. For $g \ll 1$, which has been assumed in the present paper, an allowance for this term does not influence significantly the thermoelectric power due to the two-stage drag. However, the situation $g \geq 1$ may be also realized. In the latter case, the aforementioned collision integral is important. In particular, it can lead to a much higher conductivity.

The authors are grateful to É. L. Nagaev for his discussions, critical remarks, and valuable comments.

[1)]The authors are grateful to R. N. Gurzhi for bringing this fact to their attention.

[1]C. Herring, Phys. Rev. 96, 1163 (1954).
[2]V. A. Kozlov and É. L. Nagaev, Pis'ma Zh. Eksp. Teor. Fiz. 13, 639 (1971) [JETP Lett. 13, 455 (1971)].

[3]N. S. Lidorenko, V. A. Kozlov, and É. L. Nagaev, Dokl. Akad. Nauk SSSR 204, 820 (1972) [Sov. Phys. Dokl. 17, 550 (1972)].
[4]V. A. Kozlov, N. S. Lidorenko, and É. L. Nagaev, Fiz. Tverd. Tela (Leningrad) 15, 1458 (1973) [Sov. Phys. Solid State 15, 982 (1973)].
[5]R. N. Gurzhi, Usp. Fiz. Nauk 94, 689 (1968) [Sov. Phys. Usp. 11, 255 (1968)].
[6]V. N. Kopylov and L. P. Mezhov-Deglin, Pis'ma Zh. Eksp. Teor. Fiz. 15, 269 (1972) [JETP Lett. 15, 188 (1972)].
[7]B. M. Askerov, Transport Phenomena in Semiconductors [in Russian], Moscow (1970).
[8]L. É. Gurevich and I. Ya. Korenblit, Fiz. Tverd. Tela (Leningrad) 6, 856 (1964) [Sov. Phys. Solid State 6, 661 (1964)].
[9]L. É. Gurevich and I. Ya. Korenblit, Fiz. Tverd. Tela (Leningrad) 9, 1195 (1967) [Sov. Phys. Solid State 9, 932 (1967)].
[10]A. A. Abrikosov, Zh. Eksp. Teor. Fiz. 44, 2039 (1963) [Sov. Phys. JETP 17, 1372 (1963)].
[11]J. M. Ziman, Electrons and Phonons, Clarendon Press, Oxford (1960).
[12]V. N. Kopylov and L. P. Mezhov-Deglin, Pis'ma Zh. Eksp. Teor. Fiz. 14, 32 (1971) [JETP Lett. 14, 21 (1971)].

Translated by D. Mathon