Conductivity of compensated semimetals with low carrier density in the case of phonon-phonon dragging

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It is shown that when the scattering of electrons and holes from a surface differ in character, a unique hydrodynamic conductivity mechanism is possible and leads to an exponential temperature dependence. The conditions for experimentally observing the effect in real materials are discussed.

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The purpose of the present study was to investigate the conductivity of pure semimetals such as bismuth under conditions when the conductivity is determined mainly by the dragging of electrons by phonons. The gist of this phenomenon in the indicated materials is the phonon system does not acquire, on the average, any momentum from the carriers, because the numbers of the electrons and holes are equal and the electric field causes the carriers to move in opposite directions. As a result, the dragging effect does not influence the conductivity in this case. If the scattering takes place within the electron-phonon system, the corresponding result can be rigorously justified within the framework of the kinetic equation.

On the other hand, if it is assumed that the quasiparticles can be scattered by any object, then the system is no longer balanced and a phonon flux appears. The role of such a third mechanism of scattering in very pure and perfect crystals may be assumed either by the sample surface or by an external magnetic field. As indicated recently in [1], the scattering of carriers by the surface is a very unique mechanism: the electrons are scattered almost specularly and the holes almost diffusely. The reason is the bending of the band at the surface and the formation, near the surface, of a space-charge region from which the electrons are specularly reflected, but through which the holes penetrate freely and are scattered by the surface proper. In turn, the corresponding conditions can be reversed with the aid of the field effect.

In addition, momentum can be carried away from the indicated materials by phonon-phonon umklapp processes. The physical picture corresponding to this process becomes clear from an analysis of the stage-by-stage departure of the momentum from the system. The electrons transfer the momentum acquired from the electric field to long-wave phonons, which are the only ones with which the carriers interact. The long-wave phonons transfer in N-processes the momentum to the thermal phonons (with frequencies $k_\beta T$), and the latter take the momentum out of the system with the aid of the umklapps. The non-equilibrium part of the distribution function of both the thermal and the long-wave phonons is determined in this case by the frequency of the U-processes (phonon-phonon dragging [2,3]). If the numbers of electrons and holes are not equal, this should lead to an exponential growth of the conductivity with decreasing temperature.

To understand this statement qualitatively, let us examine in detail a situation where in the electrons are reflected specularly and the holes diffusely. Since the conductivity is proportional to the sum of the paths of the carriers of both signs, it is obviously determined by the maximum of these quantities, i.e., in this case, by the electron mean free path. Under conditions of strong mutual dragging, the electrons and holes can exchange momentum via the phonons. A unique hydrodynamic situation is then produced, analogous to phonon hydrodynamics [4]. In fact, departure of the momentum from the electron system is possible in two stages: the electrons transfer the momentum to the phonons and the latter, in turn, transfer it to the holes, which take it out of the system when scattered by the surface. It is clear that the effective mean free path of the electrons should increase. Calculation shows that the changes of the order of $(l_f^+)^2/l_f^-$, where l_f^+ and l_f^- are the mean free paths of the holes and the electrons for scattering by phonons, respectively.

By the same token, competition appears between the two methods of momentum departure, namely the phonon-phonon umklapp processes and the scattering of the electrons by the surface via the long-wave phonons and holes. Since the U-processes are exponentially quenched, the conductivity can increase exponentially with decreasing temperature until the corresponding effective length becomes equalized with $(l_f^+)^2/l_f^-$. A quantitative analysis of the effect is based on the solution of the system of kinetic equations that take into account the stage-by-stage departure of the momentum. The corresponding system of equations in the absence of a temperature gradient is

$$v_z \frac{\partial f_p^{\pm}}{\partial z} + \frac{e^{\pm}}{m^{\pm}}(EP)\frac{\partial f_p^{0\pm}}{\partial \varepsilon_P} = I^N\{f_p^{\pm}, G_q\}, \tag{1}$$

$$I^N\{G_q, F_q\} + \sum_{\pm} I^N\{G_q, f_p^{\pm}\} = 0, \tag{2}$$

$$I^N\{F_q\} + I^U\{F_q\} + I^N\{F_q, G_q\} = 0 \tag{3}$$

The boundary conditions for Eq. (1) take into account the specular reflection of the holes and diffuse reflection of the electrons

$$f_p^{1\pm}(v_z; d/2) = \mathrm{P}^{\pm} f_p^{1\pm}(-v_z; d/2) \qquad v_z < 0$$

$$f_p^{1\pm}(v_z; -d/2) = \mathrm{P}^{\pm} f_p^{1\pm}(-v_z; -d/2) \qquad v_z > 0$$

where $\boldsymbol{F}_q$, and $\boldsymbol{G}_q$, are the distribution functions of the thermal and electronic phonons, respectively; $f_p^{\pm}$ is the distribution function of the holes (electrons), $f_p^{\pm}$ is its non equilibrium part, $\boldsymbol{e^{\pm}}$ is the charge of the electron (hole), and $\boldsymbol{d}$ is the sample thickness. The index "0" denotes the equilibrium part of the distribution function of the quasiparticles, "+" corresponds to holes, and "-" to electrons. The symbol $\mathbf{P^{\pm}}$ stands for $\mathbf{P}^+=1$ and $\mathbf{P}^-=0$. We seek the solution of the system (1)-(3) in the form

$$F_q = F_q^0 - (\mathbf{qu})\frac{\partial F_q^0}{\partial \omega_q}; \qquad G_q = G_q^0 - (\mathbf{qu})\frac{\partial G_q^0}{\partial \omega_q}; \qquad f_{\mathrm{P}}^{\pm} = f_{\mathrm{P}}^{0\pm} - (\mathbf{p}^{\pm}\mathbf{V}_p^{\pm})\frac{\partial f^{0\pm}}{\partial \varepsilon_p}. \tag{4}$$

The standard procedure of solving the system of kinetic equations (1)-(3) makes it possible to obtain the following expression for the conductivity $\boldsymbol{\sigma_2}$ under phonon-phonon dragging conditions:

$$\sigma_2 = \Gamma \frac{T}{s p_F}\frac{e^2 n}{\Delta_0 p_F}\left\langle\left\langle \frac{L}{L_+}\right\rangle\right\rangle\left(\frac{l^-}{l_d^-} - \frac{l^+}{l_d^+}\right)\frac{l_f^+}{\tau_{p_T p_e}} \left[\frac{1}{r^u} + \Gamma\frac{T}{s p_F}\frac{1}{r_{p_T p_e}} \times \left(\left\langle\left\langle \frac{L}{L_+}\right\rangle\right\rangle \frac{l^+}{l_d^+} + \left\langle\left\langle \frac{L}{L_-}\right\rangle\right\rangle \frac{l^-}{l_d^-}\right)\right]^{-1}. \tag{5}$$

Here $\boldsymbol{L_{\pm}}$ is the mean free path of the long-wave phonons when scattered by holes (electrons), $\boldsymbol{l^{\pm}}$ is the total mean free path of the holes (electrons), $\boldsymbol{p_F}$ *is* the Fermi momentum of the carriers, $\boldsymbol{L}$ is the total mean free path of the phonons, and $\boldsymbol{l^{\pm}}$ f is the mean free path of the carriers on the phonons. The quantity $\boldsymbol{\Delta_0}$ is given by

$$\Delta_0 = 1 - \frac{l^+}{l_f^+}\left\langle \frac{L}{L_+}\right\rangle - \frac{l^-}{l_f^-}\left\langle \frac{L}{L_-}\right\rangle.$$

The angle brackets denote averaging of the type

$$\langle \Phi(q)\rangle = \frac{1}{4p_F^4}\int_0^{2p_F} q^3 \Phi(q)dq; \qquad \langle\langle \Phi(q)\rangle\rangle = \frac{3}{8p_F^3}\int_0^{2p_F} q^3\Phi(q)dq.$$

In formula (5), furthermore, $\tau_{p_T p_e}$ and $\boldsymbol{\tau}^u$ are the relaxation times of the phonons when scattered by long-wave phonons and in *U*-processes, respectively, ***T*** is the temperature, ***s*** is the speed of sound, Ã=Ã(***s***)ξ(**s**)/Ã(4)ξ(4), Ã(***x***), and ξ(***x***) are the **γ** and **ξ** functions, $l_d^{\pm}$ is the mean free path of the carriers when scattered by the surface (equal to ***d*** in the case of diffuse reflection and to infinity in the case of specular reflection).

As follows from (5), the conductivity (σ_2 can greatly exceed the value $\sigma_0 = e^2 n/p_F\,(l^+ + l^-)$ calculated without the dragging effect, if $L_+ << L_-$ and the electrons are diffusely scattered by the surface (i.e., $l^- \approx l_d^-$) and the holes are scattered specularly (i.e., $l_d^+ \to \infty$). The conductivity increases in this case with, decreasing temperature: $\sigma_2 \approx \Gamma T / s p_F \sigma_0 \tau_u / \tau_{p_T p_e}$, reaching a maximum value $\sigma_2 \approx e^2 n (l_f^-)^2 / p_F l_f^+ >> \sigma_0$ at $l_f^- >> l_f^+$. If, to the contrary, the surface scatters the holes diffusely but, as before, $L_+ << L_-$, then the conductivity due to the dragging of the thermal phonons alters insignificantly the total conductivity $\sigma_2 \approx \sigma_0$. The latter is apparently the case in bismuth, where the dragging of the electrons by the phonons hardly changes the conductivity. However, as noted in [1], whether the carriers are reflected specularly or diffusely is determined by the bending of the band at the surface. The magnitude and sign of the bending of the bands can be altered with the aid of the field effect, changing by the same token the character of the scattering of the carriers from the surface. Therefore the conductivity of pure Bi should increase appreciably if the field effect is used to reverse the sign of the bending of the band and specular reflection of the holes from the surface is achieved. It should be noted that the condition that the dragging forces be different ($L_+ << L_-$) is not too stringent, inasmuch as the deformation potential for carriers from different bands may differ by one order of magnitude.

In turn, unbalancing the system of electrons and holes by the magnetic field has little effect on the conductivity. However, it influences substantially the thermal electric power. The corresponding expressions for the thermoelectric power $a_2(H)$ of two-step dragging at $\tau^u << \tau_{p_T p_e}$ takes the form

$$a_2(H) = \frac{l^- - l^+}{r^2} \frac{s^2 p_F}{T|e|} \tau^u \Bigg/ \left(1 + \frac{l^+ l^-}{r^2}\right), \qquad (6)$$

where ***v*** is the radius of the twisting of the carriers in the magnetic field. In strong fields $(l^{\pm}/r << 1) a_2(H)$, the value of $a_2(H)$, as expected, does not depend on the magnetic field and reaches large values at low temperatures: $a_2(H) \sim (p_F s^2 \tau^u / T|e|\, l)$. An estimate shows that in pure bismuth samples the corresponding fields are of the order of 10 Oe.

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