Hydrodynamic theory of the *s–f* exchange amplification of spin waves due to electron drift in antiferromagnets

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A self-consistent macroscopic approach which describes the *s–f* exchange amplification of spin waves in magnetically ordered crystals is developed. The amplification of spin waves by electron drift in isotropic antiferromagnets is considered for different configurations of the electric and magnetic fields, and the effects accompanying the amplification are discussed.

The possibility of amplification of spin waves in magnetically ordered crystals by fast motion of electrons was first pointed out in Ref. 1. It is important that the mechanism of the interaction of electrons with the crystal magnetization considered in Ref. 1 was relativistic, which requires the use of beams of charged particles moving at velocities close to the velocity of light.[2] The relativistic mechanism was then considered in many subsequent papers. For example, this mechanism was discussed for magnetic semiconductors in Refs. 3 and 4. The fact that the ratio v_S^2/c^2 (v_S is the phase velocity of spin waves and c is the velocity of light) which enters the expression for the gain of spin waves is small represents an obstacle to such amplification.

On the other hand, it is well known that the s–f exchange interaction of conduction electrons with the magnetization in magnetically ordered crystals is strong and may give rise to amplification which is several orders of magnitude greater than the gain due to the relativistic mechanism provided the condition $v_0 > v_S$ (v_0 is the drift velocity of electrons) is satisfied. According to Ref. 5, the amplification in crystals with an antiferromagnetic ordering, where the s–f exchange amplification mechanism is allowed by the law of conservation of momentum, is 4-5 orders of magnitude greater than the amplification due to the relativistic mechanism. Unfortunately, the microscopic approach used in Ref. 5 holds under the condition $k\ell \gg 1$ (where k is the wave vector of the spin wave and ℓ the electron mean free path), which is usually not satisfied in real crystals. The available experimental data on the mobility of carriers in magnetically ordered crystals (usually, the inequality $u \lesssim 10^2$ cm^2·V^{-1}·sec^{-1} is satisfied[6]) indicate that the opposite limiting case $k\ell \ll 1$ applies to most magnetically ordered crystals. It is our aim to describe self-consistently the amplification of spin waves in antiferromagnets due to electron drift using a macroscopic hydrodynamic approach which is asymptotically exact in the limit $k\ell \ll 1$. The Landau–Lifshitz equation for the magnetization includes not only the effective field which exists in a crystal in the absence of carriers, but also an effective field due to the s–f exchange. In this situation, our approach may be applied to describe effects in which the classical s–f exchange interaction plays an important role.

1. LANDAU–LIFSHITZ EQUATIONS FOR ANTIFERROMAGNETS TAKING INTO ACCOUNT THE s–f EXCHANGE

The evolution of the magnetization of sublattices of an antiferromagnet $\mathbf{M}_1$ and $\mathbf{M}_2$ is described in the macroscopic approach by the Landau–Lifshitz equations

$$\left.\begin{aligned} \partial\mathbf{M}_1/\partial t &= g\,[\mathbf{M}_1\times\tilde{\mathbf{H}}_1], \\ \partial\mathbf{M}_2/\partial t &= g\,[\mathbf{M}_2\times\tilde{\mathbf{H}}_2], \end{aligned}\right\} \tag{1}$$

where $g = 2\mu_0/\hbar$; $\mu_0 = e\hbar/2m_0c$ is the Bohr magneton; $\tilde{H}_1$ and $\tilde{H}_2$ are effective magnetic fields acting on the first and second sublattices. Taking into account the effect of conduction electrons, we can determine these fields by varying the total energy functional Φ of the antiferromagnet with respect to the sublattice magnetizations

$$\tilde{\mathbf{H}}_1 = -\delta\Phi/\delta\mathbf{M}_1, \quad \mathbf{H}_2 = -\delta\Phi/\delta\mathbf{M}_2 \tag{2}$$

$$\Phi = \Phi_0 + \Phi_1 \tag{3}$$

$$\begin{aligned}\Phi_0 = \int \delta_0\mathbf{M}_1\mathbf{M}_2 d^3r + \int \alpha_{12}\frac{\partial\mathbf{M}_1}{\partial x_i}\frac{\partial\mathbf{M}_2}{\partial x_i}d^3r - \frac{1}{2}\int \mathbf{H}^{(m)}(\mathbf{M}_1+\mathbf{M}_2)\,d^3r \\ - \int \mathbf{H}(\mathbf{M}_1+\mathbf{M}_2)\,d^3r + \frac{1}{2}\alpha\int\left(\frac{\partial\mathbf{M}_1}{\partial x_i}\frac{\partial\mathbf{M}_1}{\partial x_k} + \frac{\partial\mathbf{M}_2}{\partial x_i}\frac{\partial\mathbf{M}_2}{\partial x_k}\right)d^3r\end{aligned} \tag{4}$$

$$\Phi_1 = -A_s\int (M_1^z + M_2^z)\,n(r)\,d^3r \tag{5}$$

Here, Φ_0 is the energy of the antiferromagnet in the absence of conduction electron; δ_0, α, and α_{12} are phenomenological constants; $\mathbf{H}^{(m)}$ is the magnetic field induced by the magnetization of the antiferromagnet; $\mathbf{H}$ is an external magnetic field. The energy Φ_1 corresponds to the interaction of conduction electrons with the magnetization; $A_S = Aa^3/4\mu_0$, where A is the s–f exchange constant, a is the lattice constant, and n(r) is the carrier density in the conduction band.

The expression (5) corresponds to the case when all the electrons are assumed to be spin-polarized, i.e., it is assumed that the condition $\frac{1}{2}A(S_1^z + S_2^z) > \varepsilon$, holds, where $\varepsilon = \varepsilon_F$ is the Fermi energy for a degenerate gas and $\varepsilon = k_BT$ for a nondegenerate electron gas, and S is the spin of a magnetic atom.

Using Eqs. (1)-(5), we obtain the Landau–Lifshitz equations including the s–f exchange in the following form:

$$\left.\begin{aligned} \partial\mathbf{M}_1/\partial t &= g\,[\mathbf{M}_1\times\tilde{\mathbf{H}}_1] + g\,[\mathbf{M}_1\times A_s n(r)\,\mathbf{l}_z], \\ \partial\mathbf{M}_2/\partial t &= g\,[\mathbf{M}_2\times\tilde{\mathbf{H}}_2] + g\,[\mathbf{M}_2\times A_s n(r)\,\mathbf{l}_z], \end{aligned}\right\} \tag{6}$$

where $\mathbf{l}_z$ is a unit vector parallel to the z axis. It follows from Eq. (6) that the s–f exchange contribution can be regarded as an additional magnetic field if we replace in Eq. (1) the field $\tilde{H}$ by $\tilde{H} + A_S n(r, t)\,\ell_z$. Linearizing the additional terms in Eq. (6) in the vicinity of the equilibrium values of the magnetization and density, we obtain

$$
\begin{aligned}
g\,[\mathbf{M}_1 \times A_s n(r)\,\mathbf{l}_z] &= g\,[\mathbf{M}_{10} \times A_s n_0 \mathbf{l}_z] \\
&+ g\,[\mathbf{M}_{10} \times A_s n_1 \mathbf{l}_z] + g\,[\mathbf{m}_1 \times A_s n_0 \mathbf{l}_z]
\end{aligned} \tag{7}
$$

where $\mathbf{m}_1(r, t)$ and $n_1(r, t)$ are nonequilibrium corrections to the magnetization and density which should be added to the equilibrium values M_{10}, M_{20}, and n_0. It can be seen from Eqs. (6) and (7) that the first term on the right-hand side of Eq. (7) yields the following contribution to the equilibrium magnetic field:

$$\tilde{\mathbf{H}}_1^0 = -\delta_0 (\mathbf{M}_{10} + \mathbf{M}_{20}) + \mathbf{H} + g A_s n_0 \mathbf{l}_z.$$

The last term on the right-hand side of Eq. (7) leads only to a renormalization of the magnon frequencies. It follows from these results that we need to retain only the second term on the right-hand side of Eq. (7) in the linearized Landau–Lifshitz equations, provided we carry out the corresponding frequency and field renormalizations.

Considering the second term in Eq. (7)

$$g\,[\mathbf{M}_1 \times A_s n_1(r, t)\,\mathbf{l}_z] \tag{7a}$$

we can see that this term vanishes identically for a ferromagnet since $\mathbf{M}_{10} = \mathbf{M}_0 \parallel \mathbf{l}_z$ and, for an antiferromagnet, it vanishes in fields $H > H_{sf}$, where H_{sf} is the spin-flop field. According to Ref. 5, this is a consequence of the law of conservation of the total number of magnons at $T = 0$ in a ferromagnet. Using all these results, we can rewrite the system of equations (6) for the Fourier components of the field and density in the following form:

$$
\left.
\begin{aligned}
-i\omega \mathbf{m}_1 &= g\,(\mathbf{M}_{10} \times [\mathbf{h} + A_s n' \mathbf{l}_z - (\delta_0 + \alpha k^2)\,\mathbf{m}_1 - (\delta_0 + a_{12} k^2)\,\mathbf{m}_2]) \\
-i\omega \mathbf{m}_2 &= g\,(\mathbf{M}_{20} \times [\mathbf{h} + A_s n' \mathbf{l}_z - (\delta_0 + \alpha k^2)\,\mathbf{m}_2 - (\delta_0 + a_{12} k^2)\,\mathbf{m}_1])
\end{aligned}
\right\} \tag{8}
$$

2. MAGNETIC SUSCEPTIBILITY

We shall base our macroscopic description of the s–f exchange interaction of conduction electrons with a spin wave on the following expression for the force acting on an electron when a spin wave travels in the z direction in the crystal:

$$-\partial/\partial z\,(\mathcal{H}_{\text{int}})$$

where $\mathcal{H}_{\text{int}}$ is the s–f exchange Hamiltonian. We can regard this force as external (or electromotive force) which corresponds, according to Eq. (5), to the following field strength for the external forces:

$$\mathcal{E}_z = -\frac{A_s}{e}\left(\frac{\partial M_1^z}{\partial z} + \frac{\partial M_2^z}{\partial z}\right) \tag{9}$$

where e is the electronic charge. As a result, we obtain the following expression for the electrostatic induction D:

$$D_z = \varepsilon E_z + \frac{A_s \varepsilon}{e}\left(\frac{\partial M_1^z}{\partial z} + \frac{\partial M_2^z}{\partial z}\right) \tag{10}$$

where ε is the permittivity. To obtain a closed system of equations, we need to supplement Eqs. (9) and (10) by the Maxwell equations and hydrodynamic equations for the electron liquid in the medium:

$$\operatorname{curl} \mathbf{H}^{(m)} = 0, \tag{11}$$

$$\operatorname{div} \mathbf{H}^{(m)} = -4\pi \operatorname{div} (\mathbf{M}_1 + \mathbf{M}_2) \tag{12}$$

$$\operatorname{div} \mathbf{D} = -e n_1 \tag{13}$$

$$\frac{d\mathbf{v}}{dt} = \left(-\frac{e}{m}\right)(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nu \mathbf{v} + \left(\frac{T}{m}\right)\frac{1}{\rho}\nabla_r \rho, \tag{14}$$

$$\operatorname{div} \mathbf{j} + \frac{\partial \rho}{\partial t} = 0, \tag{15}$$

where $\rho = -en$ and $\mathbf{j} = -\rho \mathbf{v}$ are the charge density and the current. Equations (11) and (12) describe magnetization oscillations in the magnetostatic approximation. Equation (13) relates the induction defined by Eq. (10) to a deviation of the electron density from its equilibrium value. Equation (14) describes the drift velocity of electrons in a magnetic field taking into account collisions (ν is the electron collision rate) and the contribution of the tensor representing an internal pressure [the last term on the right-hand side of Eq. (14)]. The equation of continuity closes the system of equations (9), (10), and (11)-(15). Introducing in Eqs. (11)-(15) the Fourier components of the magnetization, density, and velocity, we obtain from Eqs. (9), (10), and (13)-(15) the following expression for the deviation of the electron density from its equilibrium value:

$$n'(k, \omega) = A_s n_0 \frac{k^2 (m_1(k, \omega) + m_2(k, \omega))}{\nu \omega_R + (\omega - k v_0)\,\mathcal{K}(\omega, k)} \tag{16}$$

$$\mathcal{K}(\omega, k) = k v_0 - \omega - i\nu + \frac{D \nu k^2}{\omega - k v_0} + \frac{\omega_c^2 \sin^2\theta\,(k v_0 - \omega - i\nu)}{\omega_c^2 \cos^2\theta - (k v_0 - \omega - i\nu)^2} \tag{17}$$

where θ is the angle between the magnetic field and the electron drift velocity; D is the diffusion coefficient of electrons, i.e., $D = k_B T / m\nu$; $\omega_c = eH/mc$ is the cyclotron frequency; $\omega_R = e^2 n_0 / \varepsilon \nu m$ is the dielectric relaxation rate of electrons. Substituting Eq. (16) in the Landau–Lifshitz equations (8), we obtain the following expressions for the components of the generalized susceptibility of an antiferromagnet including the effect of the s–f exchange:

$$
\left.
\begin{aligned}
\chi_{xx} &= \frac{\omega_+^2}{\omega_+^2 - \omega^2}\,\chi_0, \quad \chi_{yy} = \chi_0 \frac{\omega_+^2 + \omega_-^2}{\omega_+^2 - \omega_-^2} \\
\chi_{zz} &= \chi_0 \frac{\omega_-^2}{\omega_-^2 - \omega^2 + 2FG} \\
\chi_{xy} &= -\frac{i\omega\omega_+}{\omega_+^2 - \omega^2}\,\chi_0 - \chi_0^2 F_1 \frac{i\omega_+ \omega_-^2 \omega}{(\omega_+^2 - \omega^2)(\omega_-^2 - \omega^2 + 2FG)}
\end{aligned}
\right\} \tag{18}
$$

$$F = g M_0 \sqrt{1 - H^2/H_{sf}^2}\,F_1, \tag{19}$$

$$F_1 = \frac{k^2 A_s n_0 / m}{\nu \omega_R + (\omega - k v_0)\,\mathcal{K}(\omega, k)}$$

$$\omega_+^2 = (gH)^2, \quad \chi_0 = 1/\delta_0, \tag{20}$$

$$
\left.
\begin{aligned}
\omega_-^2 &= (g M_0)^2\, 2\delta_0\, (\alpha - a_{12})\, k^2\, (1 - H^2/H_{sf}^2) \\
G &= (a_{12} - \alpha)\, k^2\, (g M_0) \sqrt{1 - H^2/H_{sf}^2}
\end{aligned}
\right\} \tag{21}
$$

The expressions (18)-(21) determine completely the magnetic susceptibility tensor for an isotropic antiferromagnet taking into account the drift of electrons. The high-frequency magnetic susceptibility tensor $\hat{\chi}(k, \omega)$ is one of the fundamental quantities which determine the spin wave spectrum in a crystal, the average value and correlation function of the magnetic moment, scattering of light and neutrons by spin waves, etc. We shall now apply our results to investigate the effect of electron drift on the spin wave spectrum.

3. AMPLIFICATION OF SPIN WAVES

It follows from Eqs. (11) and (12) that the spin wave spectrum can be determined from the dispersion equation

$$k^2 + 4\pi k_i k_j \chi_{ij}(k, \omega) = 0 \tag{22}$$

Using Eqs. (18) and (22), we obtain the following dispersion equation for the magnetization which takes into account the s–f exchange:

$$(\omega_+^2 - \omega^2)[(\omega_-^2 - \omega^2) + 2FG] = 0 \tag{23}$$

It can be seen from Eq. (23) that the effect of the s–f exchange on the optical branch reduces only to a renormalization of the frequency (see Sec. 1) and does not contribute to the damping. This result was obtained in the microscopic approach in Ref. 5.

We shall now consider the case of a magnetic field parallel to the drift velocity of carriers. Using Eqs. (21), (19), and (17), we obtain from Eq. (23)

$$(\omega^2 - v_s^2k^2)(\nu\omega_R + (\omega - kv_0)(kv_0 - \omega - i\nu) + D\nu k^2) = -\bar{A}k^4v_s\nu$$
$$\bar{A} = gM_0\sqrt{1 - H^2/H_{sf}^2}\,\omega_R(\varepsilon/e)\,\Lambda_s^2\sqrt{(\alpha - \alpha_{12})/2\delta_0}$$
$$v_s = gM_0\sqrt{2\delta_0(\alpha - \alpha_{12})(1 - H^2/H_{sf}^2)} \tag{24}$$

where v_s is the phase velocity of spin waves for the branch ω_-. Equation (24) is asymptotically exact only in the limit of frequent collisions $\nu \gg \omega$, $\omega - kv_0$, i.e., in the case when the hydrodynamic description used in our approach is justified. In the limit of frequency collisions, we obtain from Eq. (24)

$$(\omega^2 - k^2v_s^2)(\omega_R - i(\omega - kv_0 + iDk^2)) = -\bar{A}k^4v_s. \tag{25}$$

Setting

$$(kv_s/\omega) = 1 + i\alpha(\omega), \quad \alpha(\omega) \ll 1, \tag{26}$$

we obtain from Eqs. (25) and (26) the following expression for the damping decrement of spin waves;

$$\mathrm{Re}\,\alpha(\omega) = -\frac{1}{2}\frac{\bar{A}\omega\gamma/v_s^3}{\left(\frac{\omega_R}{\omega}\right)^2\left(1 + \frac{\omega^2}{\omega_R\omega_D}\right)^2 + \gamma^2} \tag{27}$$

where $\gamma = (v_0/v_s - 1)$, and $\omega_D = v_s^2/D$ is the diffusion frequency.

It follows from Eq. (27), that, for $v_0 \geq v_s$, damping of spin waves changes to amplification. The expression (27) is analogous to that obtained for the amplification of sound in Ref. 4. At a fixed frequency, the gain reaches a maximum at a velocity given by

$$v^{max} = v_s(1 + \omega_R/\omega + \omega/\omega_D) \tag{28}$$

where

$$\alpha^{max}(\omega) \sim \omega^3a^3Q/v_s^3(1 + \omega^2/\omega_R\omega_D) \tag{29}$$

$$Q = A^2S\varepsilon a\sqrt{1 - H^2/H_{sf}^2}/32e^2\hbar\omega \tag{30}$$

Estimating the maximum gain from Eqs. (29) and (30) for an antiferromagnet with parameters $A = 0.5$ eV, $S = 2$, $\omega \approx 10^{11}$ sec^{-1}, $\varepsilon = 20$, $a = 3\cdot10^{-8}$ cm, and $\omega^2 \sim \omega_R\,\omega_D$, we obtain $\alpha \sim 0.1$ which clearly exceeds the possible damping of spin waves in magnetically ordered crystals by several orders of magnitude.

When the magnetic field is inclined at an angle θ to the electric field, ω_R and ω_D in Eqs. (27)-(29) should be replaced by $\omega_R' = \omega_R/\xi$ and $\omega_D' = \omega_D\,\xi$, where

$$\xi = (1 + \omega_c^2/\nu^2)/(1 + \omega_c^2\cos^2\theta/\nu^2) \tag{31}$$

In particular, we find that $\xi = 1 + \omega_c^2/\nu^2$ for a transverse field. In this case, the magnetic field plays a double role: for $\xi = 1 + \omega_c^2/\nu^2$, the drift velocity is quite independent of the mobility and is given by

$$v_0 = c|\mathbf{E}\times\mathbf{H}|/H_0^2.$$

Another important feature of the case of a transverse field is that the value of v_{max} defined by Eq. (28) corresponding to the maximum gain is reduced, which facilitates experimental observation of the effect under study.

4. CONCLUDING REMARKS

Among the most stringent conditions for the observation of drag and amplification effects of bulk spin waves by the electron drift in antiferromagnets is the Cherenkov condition. It is thus particularly important to consider effects that can identify interactions of electrons with spin waves for which the Cherenkov conditions is not necessary.[7] Among such effects we can include the drag of carriers by a spin wave, which represents an effect opposite to the effects considered earlier. The radioelectric effect is so weak in ordinary nonmagnetic crystals that it requires powerful sources of radiation to be observable. The situation is quite different in magnetically ordered crystals where the main contribution to this effect is due to the drag of carriers by a spin wave excited by an external electromagnetic wave. The magnitude of this effect in antiferromagnets can be estimated from the condition of balance between the momentum, transferred from the wave to the carriers and the resultant electric field E

$$E = (W/nev_s)\,\mathrm{Im}\,k \tag{32}$$

where W is the density of the energy flux transported by the spin wave. It follows from the results of Sec. 3 that the absorption coefficient of spin waves Im k/k in an antiferromagnet with $n \approx 10^{15}$-10^{16} cm^{-3} can reach values $\sim10^{-2}$-10^{-1}, i.e., it can be three to four orders of magnitude greater than the electronic contribution to the absorption coefficient of spin waves in ferromagnets. Setting $W = 1$ W·cm^{-2}, $n = 10^{16}$ cm^{-3}, $v_s = 10^5$ cm·sec^{-1}, and Im k $\sim$ 10 cm^{-1}, we obtain from Eq. (32) the following estimates for the strength of the resultant field: $E \sim 0.1$ V·cm^{-1}. As already noted, the main simplification in the observation of the radioelectric effect compared with the amplification effect is that it is no longer necessary to create drift currents of carriers in the sample. Hopefully, this conclusion may assist in overcoming the present difficulties in materials science that stand in the way of experimental realization of the effect of spin wave amplification in antiferromagnets.

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