NUMERICAL STUDY OF THE INTERACTION BETWEEN DUSTY PARTICLES IN A MAGNETIC FIELD

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The attraction between dusty particles in plasma placed into a magnetic field is numerically studied at various distances between the particles and various fields. The attraction force is shown to be an oscillating function of the interparticle distance in the weak magnetic field.
PACS: 52.25.Zb

The interaction of two dusty particles in plasma placed in a magnetic field was qualitatively analyzed in [1]. It was shown that in the limit of weak magnetic field the attraction between dusty particles, caused by bombardment with the plasma ion component, becomes especially strong at distances multiple to $2\pi R$, where R is the Larmor radius of ions. In the case of an intense field, the interaction between dusty particles becomes maximum and independent of the field. In the present work we numerically study the model considered in [1].

That model considers the dusty particle as a point isotropic source of charged particles (ions), placed into a uniform constant magnetic field $\boldsymbol{H}$. It is assumed that the interaction between ions and their collisions with the neutral component (collisionless plasma) can be neglected and all emitted ions have the same velocity $\boldsymbol{V}$.

In such a statement the problem is equivalent to the initial problem of electron (ion) optics where the uniform magnetic field is used for imaging [2]. Since in the theory of electron-optical systems main attention is paid to charged particle beam focusing and qualitative images, its scope is as a rule limited by the scale of focal length. In contrast, to describe the interaction of dusty particles, the lengths, which may greatly exceed the distance to the first focus are vital.

Further we consider that one particle is at a given distance l from another particle of the radius α which is much less than l. A line connecting dusty particles is directed along the magnetic field. All ions hitting the target are assumed to stick in it.

When calculating the momentum transmitted to the second particle per unit time, the first dusty particle is believed to be at the coordinate origin and the magnetic field direction corresponds to the axis **z**. Then ions with an orbit (screw line) radius shorter than **a/2** inevitably hit the second particle independently of l. This is the case if the emission angle θ (measured from the axis **z**) is sufficiently small. Other ions emitted from the origin periodically return to the axis z and can or cannot hit the second particle depending on the distance l.

At fixed l and a not too strong field, there exist an infinite number of angle θ sectors, for which an ion hits the target

$$0 \le \theta \le \theta_0,\ \theta_1 \le \theta \le \theta_2,\ \theta_3 \le \theta \le \theta_4$$

etc., where θ_k are concentrated at $\theta_k = \pi/2$, while the sectors obviously decrease.

We put $c=a/2R$ and $z = l/2R$ and consider the second particle as a disk of radius a, orthogonal to the axis z with a center at it. The momentum transferred to the second particle per unit time that is the attraction force F between the particles is given by

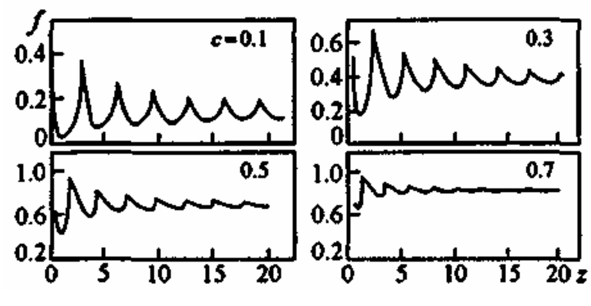

Figure 1. Dependence f(**z**) tor various parameters $c=a/2R$.

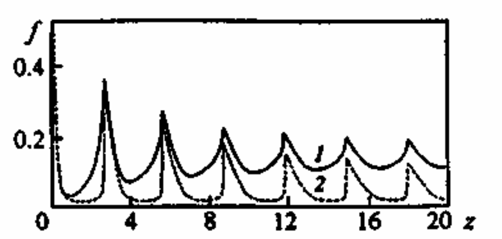

Figure 2. Comparison of $f(z)$ (1) to $f_0(z)$ (2) at $c=0.1$.

$$F(z)=1/4\ mVI\ f(z) \tag{1}$$

where m and V are respectively the ion mass and velocity, I is the number of ions emitted by the first dusty particle per unit time.

At $c \geq 1$ the force is independent of the distance, $f(z) \equiv 1$. According to [1], at $c < 1$ the function f is given by the infinite series

$$f(z)=A_0^2 + (A_2^2 - A_1^2) + (A_4^2 - A_3^2) + \dots, \tag{2}$$

where $A_k = sin\ \theta_k$ are the roots of the equation

$$c = A \left| sin \frac{z}{\sqrt{1-A^2}} \right| \tag{3}$$

When writing series (2), it is believed that A_k (or θ_k) are indexed in ascending order.
Calculation of $f(z)$ is mainly complicated by a great number N of series (2) terms: N should exceed $z/\pi\sqrt{\varepsilon}$ to determine confidently the value of $f(z)$ with an error smaller than ε. For instance, for $\varepsilon = 0.001$ and $z = 20$, N should exceed 200. The computing procedure is outlined elsewhere [3] in detail.

The calculated values $f(z)$ are displayed for some c in Fig. 1. Functions $f(z)$ are no monotonic at any $c<1$. Each $f(z)$ has an infinite number of maxima and minima, corresponding to the attraction extrema between dusty particles. The amplitude of oscillations, that is the difference between neighboring maxima and minima, decreases as z grows. As $z \to \infty$, $f(z)$ tends to the constant

$$F_\infty = \frac{mVI}{4}\left(1 + \frac{2}{\pi} c\sqrt{1-c^2} - \frac{2}{\pi} cos^{-1} c\right). \tag{4}$$

Figure 2 shows $f(z)$ calculated at $c = 0.1$ in comparison to $f_0(z) = sin^2\ \theta_0(z)$ used in [1] in the limiting case of a weak magnetic field (the first term of series (2)). This comparison indicates the use of $f_0(z)$ to be justified only for several first oscillations in the attraction force. It is noteworthy that $f_0(z)$ is discontinuous while $f(z)$ is continuous, however, with a singular derivative at discontinuity f_0 points. We note also that the statement of [1] about the maximum attraction at distances l multiple to $2\pi R$ (z multiple to π) is approximate. For instance, at $c = 0.1$ the first maximum of $f(z)$ is at the point $z_0 = 2.68$ rather than at $z = \pi$. The asymptotic formula $z_0 \approx \pi - 3/2\pi^{1/3}c^{2/3}$ for small c yields the maximum position at $c = 0.1$ with a good accuracy.

The value $f(z)$ can be found using the SUM1.FOR program (available at the Institute of Mathematical Problems of Biology) by the given argument z and parameter c.

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