INTERACTION BETWEEN DUSTY PARTICLES OF COLLISIONAL PLASMA IN A MAGNETIC FIELD

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Formulas are derived for the Lesage forces between dusty particles in collision plasma placed into magnetic field. The attraction force is shown there to have a form of damped oscillations. Conditions of binding dusty particles are discussed. PACS: 52.25.Zb

Physics of dusty plasma is a new and rapidly developing line of inquiry (see [1-3]). Dusty-plasma crystals discovered in a rarefied low-temperature plasma of gas discharge have made especially urgent the investigation of interaction between dusty particles. Various dusty plasma examples are given in [1, 2]. Currently, dusty crystals are found only in low-pressure collisions plasma. The impact of an external magnetic field on binding between two such particles was considered in [4, 5]. In the absence of field, equilibrium distances between dusty particles are controlled by competition of two factors: the Coulomb repulsion and an attraction due to bombardment of particles by the ion component. According to [4, 5], the external magnetic field bends ion trajectories so that the interaction between dusty particles is maximum at distances multiple to $2\pi R$, where R is the Larmor radius.

In the present work we consider the impact of ion collisions with each other and with the plasma neutral component in a magnetic field on the interaction of dusty particles. For the frequent collisions when

$$d \gg l_0 , \tag{1}$$

(where d is the distance between dusty particles and l_0 is the free path-length of bombarding particles), the attraction forces caused by bombardment become additive. Therefore, the results found for two particles can be applied to a great number of particles, for instance, to a dusty-plasma crystal. If such a crystal is formed due to attraction of bombarded dusty particles, these forces should be stronger in high-pressure plasma, since the effect is proportional to the number of bombarding particles. On the other hand, the attraction exponentially falls as the distance increases in the collision case.

To calculate the above attraction, we invert the problem as in [4, 5], considering an ion hitting a particle to be emitted rather than absorbed. These ions bombard another particle, thus causing repulsion between the emitting and the target particles. The repulsion force taken with an opposite sign is just the sought-for attraction force between dusty particles. If the distance between them much exceeds their radius, the emitting dusty particle is considered a point particle.

Also similar to [4, 5], we assume ions emitted by the point particle to have the same velocity **V**. All the ions move along screw trajectories starting from the emitting particle. We designate the number of ions emitted by it per unit time as $I(0)$. The number $I(L)$ of ions crossing the surface passing a screw path L is given by

$$I(L) = I(0)\, exp\, (-L/l_0) \ . \tag{2}$$

The number dI_θ of ions emitted per unit time in the sector $(\theta, \theta + d\theta)$, where θ is the angle between the external magnetic field and the particle velocity, is given by

$$dI_\theta = [1/2\, I(0)]\, sin\ \theta d\theta . \tag{3}$$

Taking into account the relation $r_L = mcV \times \sin\theta/qH$ between the ion velocity and the Larmor radius, where m is the ion mass and H is the magnetic field, formula (3) is rewritten as

$$d\, I_\theta = \frac{I(0)}{2} \frac{1}{\sqrt{1 - r_L^2/R^2}} \frac{r_L}{R^2} \ , \tag{4}$$

where $R = mcV/qH$.

Let the field be directed parallel to the axis z. Then if the particle is in the coordinate origin, an ion emitted by it at the angle θ crosses the plane, perpendicular to the magnetic field and spaced from the origin by a distance z, at the length

$$\rho = 2r_L \left| \sin\left(\frac{z}{2R\sqrt{1-r_L^2/R^2}} \right) \right| \tag{5}$$

from the axis.

Ions emitted within the sector (θ, $\theta + d\theta$) hit the unit area $2\pi\rho\, d\rho$ of the considered plane. The fraction of ions hitting this area without collisions is defined by (4) where $I(0)$ is replaced with $I(L)$ defined by (2) and L is the trajectory length to the area $2\pi\rho\, d\rho$. Thus, according to (4) and (5), the number $\partial I/\partial S$ of ions crossing the unit area per unit time is given by

$$\frac{\partial I}{\partial S} = \frac{I(L)}{2} \frac{1}{\sqrt{1-r_L^2/R^2}} \frac{r_L}{R^2} \frac{1}{2\pi\rho\,(\partial\rho/\partial r_L)}, \tag{6}$$

where dp/dr_L is determined by (5). In (6), the length L of a trajectory crossing the plane perpendicular to the magnetic field and spaced from the dusty particle by the distance z and by the distance ρ from the axis is related to z as

$$L = L(r_L, z) = \frac{z}{\sqrt{1-r_L^2/R^2}}. \tag{7}$$

We consider the second particle of radius o to be at the same distance z from the first particle. When all ions are absorbed by the dusty particle, the force F caused by its ion bombardment is given by

$$F = \int_0^a 2\pi m V_{\|} \frac{\partial I}{\partial S} \rho\, d\rho, \qquad 2R \geq a, \tag{8}$$

$$F = \int_0^{2R} 2\pi m V_{\|} \frac{\partial I}{\partial S} \rho\, d\rho, \qquad 2R < a, \tag{9}$$

where $V_{\|}$, is the ion velocity component parallel to the magnetic field.

Substitution of (6) into (8) and (9) yields the force F in the case of $2R \geq a$,

$$F = \frac{I(0) mV}{2R^2} \int_0^{\tilde{a}_0} r_L \exp\left[-\frac{L(r_L, z)}{l_0} \right] dr_L + \frac{I(0) mV}{2R^2} \sum_{i=1}^{\infty} \int_{\tilde{a}_{2i-1}}^{\tilde{a}_{2i}} r_L \exp\left[-\frac{L(r_L, z)}{l_0} \right] dr_L \tag{10}$$

where $\tilde{a}_i$ are determined by the formula

$$a = 2\tilde{a}_i \left| \sin\left(\frac{z}{2\sqrt{R^2 - \tilde{a}_i^2}} \right) \right|. \tag{11}$$

In (10) and (11) we number the roots $\tilde{a}_i$ in ascending order. If $2R < a$, one has

$$F = \frac{I(0) mV}{2R^2} \int_0^{R} r_L \exp\left[-\frac{L(r_L, z)}{l_0} \right] dr_L. \tag{12}$$

When the value $L(r_L, z)$ is defined by (7), formula (10) is written as

$$F=\frac{I(0)mV}{2}\int_{1}^{(1-\tilde{a}_0^2/R^2)^{-1/2}}\frac{\exp(-zx/l_0}{x^3}dx+\frac{I(0)mV}{2}\sum_{i=1}^{\infty}\int_{(1-\tilde{a}_{2i-1}^2/R^2)^{-1/2}}^{(1-\tilde{a}_{2i}^2/R^2)^{-1/2}}\frac{\exp(-zx/l_0)}{x^3}dx,\ \ 2R\geq a. \tag{13}$$

Correspondingly, from (12) one finds

$$F=\frac{I(0)mV}{4}\left[-\left(\frac{z}{l_0}\right)^2 E_i\left(-\frac{z}{l_0}\right)+\left(1-\frac{z}{l_0}\right)\exp\left(-\frac{z}{l_0}\right)\right],\ \ 2R<a, \tag{14}$$

where $E_i(z)$ is the integral exponential function. It follows from (14) that in the strong magnetic field, F is independent of it. This result is physically clear. Since the particle radius α exceeds the maximum diameter of Larmor's ion rotation circle, all ions emitted by the first particle are absorbed by the second one.

In the limiting case of weak magnetic fields, when $R \gg a$, from (13) and (11) one derives which

$$F=\frac{I(0)mV}{16}\frac{a^2}{R^2}\frac{e^{-z/l_0}}{\sin^2(z/2R)}, \tag{15}$$

indicates that the resonant attraction in the field is retained in the case of collisions. Formula (15) is valid on the whole axis **z**, except for ranges ($2\pi Rn - a$, $2\pi Rn + a$) where n = 1, 2, 3, ... In the collisional case, the attraction force represents damped oscillations with period 2nR. In the limiting case of $z \ll R$, it follows from (15) that

$$F=\frac{I(0)mV}{4}\frac{a^2}{z^2}\exp\left(-\frac{z}{l_0}\right) \quad . \tag{16}$$

Thus, when the distance between dusty particles is much shorter than the Larmor radius, force (16) exactly coincides with the attraction force between dusty particles with collisions but without magnetic field and has the form of a screened Coulomb force. In the opposite case of the strong field when $2R< a$, it follows from (14) that

$$F=\frac{I(0)mV}{2}\frac{l_0}{z}e^{-z/l_0}, \qquad z \gg l_0 . \tag{17}$$

As was indicated above, to date dusty-plasma crystals have not been observed in thermal plasma. If the crystalline structure is controlled by the attraction, at first glance, such forces should be stronger in high- than in low-pressure plasma since the bombardment force is proportional to the number $I(0)$ of bombarding particles. It follows from (16) that when the electrostatic repulsion between dusty particles is Debye's, in the case of $r_D > l_0$ (where r_D is the Debye radius), the repulsion exceeds attraction and the bound state of dusty particles is impossible in the absence of magnetic field. Since $l_0 \approx n^{-1}$ (where n_0 is the neutral component concentration) and $r_D \approx (\alpha n_0)^{-1/2}$ (where α is the plasma ionization degree), the condition $r_D > l_0$ should always be satisfied as n_0 grows, beginning from a certain value, which just breaks the bond between dusty particles. When condition (1) is satisfied, all the aforesaid relates to the dusty crystal.

The important role of the magnetic field is that it strengthens attraction between dusty particles arranged along lines of force to stabilize the crystalline structure in this direction. According to (15) and (16), in the weak field ($R \gg \alpha$) the ratio of the attraction force $F(H)$ to the force $F(0)$ in the absence of field for two dusty particles spaced by the resonant distance $2\pi R$ is proportional to $(R/\alpha)^{4/3}$ and is independent of the free pathlength l_0. Thus, when both condition (1) and the inequality $l_0 > r_D$ are satisfied, the dusty crystal can be rebuilt into a crystal with a period close to $2\pi R$ along the magnetic field.

Finally, we note that the experiments on the impact of magnetic field on dusty particles have not yet been carried out. At the same time, dusty-plasma structures seem to be produced in the field induced to confine plasma in thermonuclear setups and in the near-wall plasma (see [6, 7]). To study such structures in operating setups one needs corresponding diagnostic equipment.

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