On the term of the 4-th order with respect to the field operators in the translation-invariant polaron theory

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Abstract

It is shown that 4-th order term in the translation-invariant polaron theory vanishes.

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Having radically changed the concept of polarons, the theory of translation-invariant polarons (TI-polarons) [1]-[2] has recently came into focus of attention [3]-[8]. In this connection we discussed this theory in detail in review [9]. Comments on papers [3]-[9], that have come to the author suggest that most questions are concerned with vanishing of the contribution into the TI-polaron ground state energy made by the term of the 4-th order with respect to the field operators which arises in Froehlich Hamiltonian after Lee, Low, Pines (LLP) transformation [10] (Appendix 1 in [9]). Though the proof of this statement is given in [1], [11] and in [9], it seems not to be explicit enough, some details are omitted. The aim of this paper is to discuss the point in detail.

According to [9], the term of the 4-th order with respect to the phonon field operators $H_1^{(4)}$ has the form:

$$H_1^{(4)} = \frac{1}{2m} \sum_{k,k'} \vec{k}\vec{k}' a_k^+ a_{k'}^+ a_k a_{k'} \tag{1}$$

Accordingly, the contribution of the term $H_1^{(4)}$ into the ground state energy

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is:

$$E_1^{(4)} = \sum_{k,k'} \vec{k}\vec{k'} \rho_{\vec{k},\vec{k'}} \tag{2}$$

$$\rho_{kk'} = \langle 0|\Lambda_0^+ a_k^+ a_{k'}^+ a_k a_{k'} \Lambda_0|0\rangle \ ,$$

$$\Lambda_0 = C \exp\left(\frac{1}{2}\sum_{k,k'} a_k^+ A_{kk'} a_{k'}^+\right) ,$$

where $A_{kk'}$ is a symmetrical matrix: $A_{kk'} = A_{k'k}$. It is easy to see that:

$$a_{k'}\Lambda_0 = \sum_{k''} A_{k'k''} a_{k''}^+ \Lambda_0 \tag{3}$$

Therefore:

$$\Lambda_{k,k'} = a_k a_{k'} \Lambda_0 = A_{k'k}\Lambda_0 + \sum_{k'',k'''} A_{k'k''} A_{kk'''} a_{k''}^+ a_{k'''}^+ \Lambda_0 \tag{4}$$

Hence, function $\rho_{\vec{k},\vec{k'}}$ in (2) is the norm of the vector $\Lambda_{k,k'}$:

$$\rho_{kk'} = \langle 0|\Lambda_{kk'}^+ \Lambda_{kk'}|0\rangle \tag{5}$$

Let us show that the matrix $A_{kk'}$ has the structure:

$$A_{kk'} = (\vec{k}\vec{k'}) Q(|\vec{k}|, |\vec{k'}|) \tag{6}$$

For this purpose let us use equation (7.7) from [9] determining functional of the ground state Λ_0:

$$\left(\sum_{k'} M_{1kk'}^* a_{k'} - \sum_{k'} M_{2kk'}^* a_{k'}^+\right) \Lambda_0|0\rangle = 0 \tag{7}$$

With the use of (3) and (7) we get the condition:

$$\sum_{k'} M_{1kk'}^* A_{k''k'} - M_{2kk''}^* = 0 \tag{8}$$

According to [1], [2], matrix $M_{1,2kk'}$ has the structure: $M_{1,2kk'} = (\vec{k}, \vec{k'}) R_{1,2}(|\vec{k}|, |\vec{k'}|)$. Hence, in accordance with condition (8) matrix A (6) has the same structure. From (4)-(6) immediately follows that

$$\rho_{\vec{k},\vec{k'}} = \rho_{-\vec{k},\vec{k'}} = \rho_{\vec{k},-\vec{k'}} \tag{9}$$

and $E_1^{(4)}$ (2) becomes zero which was to be proved.

Notice that if the total momentum of a TI-polaron $\vec{P}$ is nonzero, then matrix A no longer has the structure of (6): multiplier Q in this case becomes angular dependent. Expression for the ground state energy $E(P)$ given in [9] is valid in this case only in the limit $\vec{P} \to 0$.

In conclusion the author would like to thank Prof. Devreese for his recommendation to present proofs of [1], [2], [9] in greater detail.

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References

[1] Tulub A.V. *Vestnik Leningrad Univ.* **22** 104 (1960)

[2] Tulub A.V. *Sov. Phys. JETP* **14** 1301 (1962)
`http://www.jetp.ac.ru/cgi-bin/e/index/r/41/6/p1828?a=list`

[3] Lakhno V D *JETP* **110** 811 (2010)

[4] Lakhno V D *Sol. St. Comm.* **152** 621 (2012)

[5] Lakhno V D *JETP* **116** 892 (2013)

[6] Klimin S N, Devreese J T *Sol. St. Comm.* **152** 1601 (2012)

[7] Klimin S N, Devreese J T *Sol. St. Comm.* **153** 58 (2013)

[8] Kashirina N.I., Lakhno V.D., Tulub A.V. *JETP* **114** 867 (2012)

[9] Lakhno V.D. *Phys.-Usp.* **58** 295 (2015); arXiv:1505.01699

[10] Lee T.D, Low F.E, Pines D. *Phys. Rev.* **90** 297 (1953)

[11] Röseler J *Phys. Status Solidi* **25** 311 (1968)