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SIZE OF THE DUSTY PLASMA MOLECULE IN A MAGNETIC FIELD

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We consider a dependency of the equilibrium distance between dusty particles in Maxwellian plasma on an external magnetic field in the absence of gravity.

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Equilibrium states of the dusty molecule in low-temperature plasma placed in a magnetic field were considered in [1]. It was believed that the equilibrium of charged dusty particles is controlled there by a balance of the Coulomb repulsion and the attraction caused by ion bombardment. When calculating the latter force, the ion velocities were assumed to be identical by absolute values.

This work is aimed to calculate the attraction force of dusty particles in an equilibrium plasma with the Maxwell distribution of ions.

In the absence of gravity and for identical ion velocities V, the Lesage attraction force F_L caused by ion bombardment for two dusty particles occurring at a line parallel to the magnetic field is given in [2] as

$$F_L = -\frac{mIV}{4}\gamma(l,a,R), \quad \frac{a}{2R} \leqslant 1, \qquad (1)$$

where I is the number of ions incident onto the particle of radius a per unit time, l is the distance between dusty particles, R is the ion Larmor radius, and m is the ion mass.

We note that the function $\gamma(l,a,R)$ (tabulated in [3]) depends on two substantial variables, $\gamma(l,a,R) = \gamma(l/2R, a/2R)$. At $a/2R \geqslant 1$ the force is independent of distance so that $\gamma \equiv 1$.

The force $\overline{F}_L$ of ion bombardment with the Maxwell distribution $f(V)$ of ion velocities is written as

$$\overline{F}_L = \int F_L(R) f(R)\, d^3R, \qquad (2)$$

where $f(R)$ is the distribution over Larmor radii, derived from the Maxwell one using the relationship $V = \omega_H R$ (ω_H is the Larmor frequency of ions),

$$f(R) = \omega_H^3 (m/2\pi T)^{3/2} \exp(-m\omega_H^2 R^2/2T), \qquad (3)$$

T is the ion temperature.

Equations (1)–(3) yield the formula for $\overline{F}_L$

$$\overline{F}_L = \sqrt{\pi}\, T n_i a^2 F(t, x_0), \qquad (4)$$

$$F(t,x_0) = 2x_0^5 \int_0^1 dx\, x^{-6}\gamma(t,x)\exp\left(-\frac{x_0^2}{x^2}\right) + \int_0^{x_0} dx\, x^4 \exp(-x^2), \qquad (5a)$$

$$x_0 = \frac{1}{2}\left(\frac{m}{2T}\right)^{1/2}\omega_H a, \quad t = \frac{l}{a}, \qquad (5b)$$

where n_i is the ion concentration. According to [3], the function $\gamma(t,x)$ is given by the infinite series

$$\gamma(t,x) = A_0^2 - (A_1^2 - A_2^2) - (A_3^2 - A_4^2) - \cdots \qquad (6)$$

where A_r are roots of the equation

$$x = A\left|\sin\frac{tx}{\sqrt{1-A^2}}\right| \qquad (7)$$

In the accepted notation, the roots of (7) follow in ascending order.

Figure 1 shows dependency of the attraction force between dusty particles on the distance l

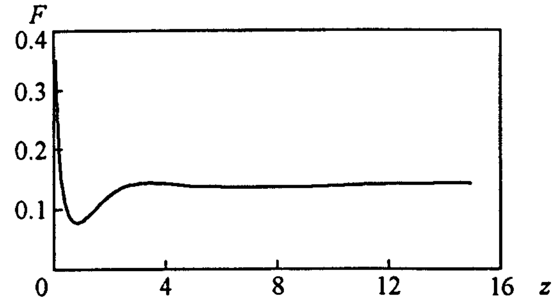

Figure 1. dependency of the attraction force $F_L = \sqrt{\pi}\, T n_i a^2 F$ of dusty particles on the distance $l = za/x_0$ at $x_0 = 0.228$.

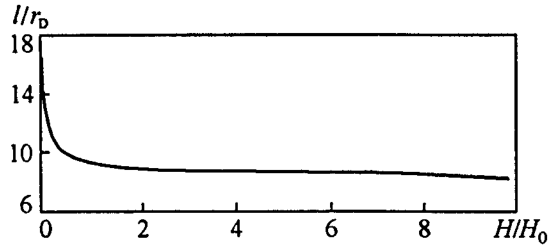

Figure 2. dependency of the equilibrium distance l between dusty particles on the magnetic field H at $H_0 = 10^4$ Oe. See other parameters in the text.

for the value $x_0 = 0.228$ corresponding to the plasma parameters $a = 10\,\mu$m, $n_i = 10^9\,\text{cm}^{-3}$, $T = 290$ K, and $H = 10^4$ Oe.

To calculate the equilibrium positions of dusty particles, one should equate the attraction force F_L (4) to the Coulomb repulsion force F_D. The latter between the analogously charged particles is controlled by the Debye screening and written as

$$F_D = \frac{q^2}{l}\left(\frac{1}{l} + \frac{1}{r_D}\right) \exp\left(-\frac{l}{r_D}\right), \qquad (8)$$

where q is the dusty particle charge and $r_D = (T/4\pi n_i e^2)^{1/2}$ is the Debye radius.

Using (4) and (8), the condition of dusty particle equilibrium takes on the form

$$\frac{1}{z}\left(\kappa + \frac{1}{z}\right) \exp(\kappa z) = \eta F\left(\frac{z}{x_0}, x_0\right), \qquad (9)$$

where

$$\kappa = \frac{2}{\omega_H r_D}\left(\frac{2T}{m}\right)^{1/2}, \quad \eta = 8\sqrt{\pi}\,\frac{T^2 n_i a^2}{q^2 \omega_H^2 m}, \quad \frac{l}{r_D} = \kappa z. \qquad (10)$$

For physically interesting parameters κ and η, Eq. (9) has a single solution.

Figure 2 displays dependency of the equilibrium distance between dusty particles on the magnetic field for $q = 10^4 e$, $H_0 = 10^4$ Oe, and the plasma parameters taken above: $\kappa = 0.816$, $\eta = 5.9 \cdot 10^{-5}$, and $r_D = 3.7 \cdot 10^{-3}$ cm. It is evident that the external magnetic field in the range $0 < H < 10^4$ Oe affects strongly the equilibrium distance between particles. At $H = 10^4$ Oe the dusty molecule size appears almost half less than that in the absence of field.

References

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