Amplification of electromagnetic waves by Cherenkov electrons in uniaxial antiferromagnets in strong magnetic fields

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(Submitted October 25, 1984; resubmitted March 18, 1985)
Fiz. Tverd. Tela (Leningrad) 27, 2920–2925 (October 1985)

Calculations are made of the gain in the amplification of spin waves by Cherenkov electrons in an anisotropic antiferromagnet subjected to a strong magnetic field. Conditions for the amplification of electromagnetic waves are formulated. Compared with ferromagnets, the gain in antiferromagnets is 4–5 orders of magnitude greater and this facilitates an experimental investigation of the effect.

The most promising approach to the problem of development of amplifiers of electromagnetic waves at millimeter and shorter (up to tens of microns) wavelengths is based on the amplification of spin waves in magnetically ordered crystals.

A possible mechanism of such amplification and excitation of spin waves is the Cherenkov excitation of magnons by fast electrons. Such a mechanism was studied in Refs. 1 and 2 for ferromagnets. It is important to note that only the relativistic interaction of particles with the crystal magnetization contributes to the amplification of spin waves in this case because of the conservation of the total spin. This fact makes it necessary to use electrons with velocities close to the velocity of light.[1]

The situation is quite different for carriers in antiferromagnets. It was shown in Ref. 3 that the s–f exchange mechanism of creation of magnons is not forbidden in antiferromagnets, which opens up a way of excitation of spin waves which is particularly effective in a magnetic field. The amplified spin waves transform into electromagnetic waves near the antiferromagnetic resonance. In an isotropic antiferromagnet, such a transformation is hindered since carriers excite acoustic magnons, whereas electromagnetic waves at an antiferromagnetic resonance interact with the optical branch. It is thus of practical interest to calculate the gain in the amplification of spin waves by carriers in anisotropic antiferromagnets where an antiferromagnetic resonance takes place for both branches of the magnetization oscillations. We shall consider the case of an antiferromagnet with a uniaxial anisotropy.

1. HAMILTONIAN OF THE s–f EXCHANGE INTERACTION IN UNIAXIAL ANTIFERROMAGNETS

A Hamiltonian describing the motion of conduction electrons in anisotropic antiferromagnets in a magnetic field is, in general, given by[4]

$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{int} + \mathcal{H}_M$$

$$\mathcal{H}_e = \frac{1}{2m^*}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2, \quad \mathcal{H}_{int} = -\sum_{m, m'} A(R - R_{m'})(S_{m'}, s_{m'})$$

$$\mathcal{H}_M = -\frac{1}{2}\sum_{i, m_1 \neq m_1'} I^i_{m_1 m_1'} S^i_{m_1} S^i_{m_1'} - \frac{1}{2}\sum_{i, m_2 \neq m_2'} I^i_{m_2 m_2'} S^i_{m_2} S^i_{m_2'} - \sum_{i, m_1, m_2} I^i_{m_1 m_2} S^i_{m_1} S^i_{m_2} - \sum_{i, m_1} S^i_{m_1} H_i - \sum_{i, m_2} S^i_{m_2} H_i \qquad (1)$$

where $\mathcal{H}_e$ describes electron motion in a magnetic field with vector potential **A**; $\mathcal{H}_{int}$ corresponds to the interaction of conduction electrons with the magnetic subsystem of a crystal characterized by an s–f exchange interaction constant $A_{mm'} = A\delta_{mm'}$; $\mathcal{H}_M$ is the exchange Hamiltonian for an anisotropic antiferromagnet subjected to a magnetic field H (H is measured in energy units). For a uniaxial antiferromagnet with the anisotropy axis parallel to the y axis, the exchange integrals are given by

$$I^x_{mm'} = I^z_{mm'} = I_{mm'}, \quad I^y_{mm'} = I_{mm'} + \Delta I_{mm'} \qquad (2)$$

Restricting ourselves to the interaction between two equivalent sublattices, we shall introduce the following notation:

$$\sum_{m_1, m_2} I_{m_1 m_2} S^2 = N J_{12}, \quad \sum_{m_1, m_2} \Delta I_{m_1 m_2} S^2 = N \Delta J_{12} \qquad (3)$$

where N is the number of magnetic atoms per sublattice.

We shall consider an external magnetic field H parallel to the z axis, i.e., perpendicular to the anisotropy axis. In this case, the magnetization vector an antiferromagnet is parallel to the field in the whole range of H ($\Delta J_{12} < 0$). Assuming that the field is strong enough for the electron spin to be completely polarized in the direction of the field and introducing the magnon creation and annihilation operators ξ^+ and ξ in $\mathcal{H}_M$ and $\mathcal{H}_{\rm int}$ and the electron creation and annihilation operators a^+ and a in $\mathcal{H}_e$ and $\mathcal{H}_{\rm int}$ we can transform Eq. (1) to the form

$$\mathcal{H} = \mathcal{H}_e + \mathcal{H}_{\rm int}^{(1)} + \mathcal{H}_{\rm int}^{(2)} + \mathcal{H}_M,$$
$$\mathcal{H}_e = \sum_\alpha \varepsilon_\alpha a_\alpha^+ a_\alpha, \quad \mathcal{H}_M = \sum_{j,q} \hbar\omega_{qj} \xi_{qj}^+ \xi_{qj}$$
$$\mathcal{H}_{\rm int}^{(1)} = \sum_{\alpha\alpha' q} A^{(j)}(\alpha\alpha' q)\, a_\alpha^+ a_{\alpha'} (\xi_{-qj}^+ + \xi_{qj})$$
$$\mathcal{H}_{\rm int}^{(2)} = \sum_{\alpha\alpha' qq' j} B^{(j)}(\alpha\alpha' qq')\, a_\alpha^+ a_{\alpha'} \xi_{-qj}^+ \xi_{q'j}$$
$$+ \sum_{\alpha\alpha' qq' j} C^{(j)}(\alpha\alpha' qq')\, a_\alpha^+ a_{\alpha'} (\xi_{-qj}^+ \xi_{-q'j}^+ + \xi_{qj}\xi_{q'j}) \tag{4}$$

where j (=1, 2) corresponds to the two magnon branches in an anisotropic antiferromagnet. The matrix elements of A, B, and C are given by

$$\left.\begin{aligned} A^{(j)}(\alpha, \alpha', q) &= Q_q \langle \alpha | e^{iqr} | \alpha' \rangle \delta_{j,2}, \\ B^{(j)}(\alpha\alpha' qq') &= \Gamma_{qq'}^{(j)} \langle \alpha | e^{i(q+q')r} | \alpha' \rangle, \\ C^{(j)}(\alpha\alpha' qq') &= Z_{qq'}^{(j)} \langle \alpha | e^{i(q+q')r} | \alpha' \rangle. \end{aligned}\right\} \tag{5}$$

The index α in Eqs. (4) and (5) labels the eigenfunctions of the kinetic energy operator of an electron in a magnetic field $\mathcal{H}_e$ [with the eigenvalues $\varepsilon_\alpha = p_z^2/2m^* + \hbar\Omega\left(n + \frac{1}{2}\right)$ and it includes the quantum numbers n, p_z, p_y, where n is the number of the Landau level and p_z and p_y are the components of the electron momentum in the corresponding directions (we shall use the gauge $A_x = 0$, $A_y = Hx$, and $A_z = 0$). In particular, using our model, we find that the coefficients C, Γ, and Z in Eq. (5) have the following form for j ≠ 0:

$$Q_q = A(SH_1^*)^{1/2}(1 - H^2/H_E^2)^{3/4}/4N(H_E(1 - H^2/H_E^2) + 2H_a)^{1/2},$$
$$\Gamma_{qq'} = AH(H_E(1 - H^2/H_E^2) - H_a)/8NH_E H_1 (1 - H^2/H_E^2)^{1/2},$$
$$Z_{qq'} = -AH(H_E(1 - H^2/H_E^2) + H_a)/16NH_E H_1 (1 - H^2/H_E^2)^{1/2},$$
$$H_1 = (H_E H_a)^{1/2}, \tag{6}$$

where H_a is the anisotropy field and H_e is the spin-flop exchange field. We have included in Eq. (6) only the lead-in terms of the wave vector of magnetization oscillations. In our geometry the magnon frequencies have the following form in the same approximation:

$$\omega_{q1} = \mu(H^2 + H_{EA}^2)^{1/2} \tag{7}$$

$$\omega_{q2} = \mu H_{EA}(1 - H^2/H_E^2)^{1/2} \tag{8}$$

where μ is the atomic magnetic moment and $H_{EA} \sim H_\parallel$. The fields H_a and H_E which appear in Eqs. (6) and (8) can be expressed in terms of the exchange integrals (2) and (3) and, in the case $|\Delta J_{12}| \ll |J_{12}|$, they are given by

$$H_E \approx 4|J_{12}|, \; H_a \approx 2|\Delta J_{12}|$$

2. AMPLIFICATION OF SPIN WAVES IN A MAGNETIC FIELD

A transport equation for the magnon distribution function describing one- and two-magnon emission and absorption events corresponding to the Hamiltonian (4) is given by

$$\frac{\partial m_q^{(j)}}{\partial t} = \frac{2\pi}{\hbar} \sum_{\alpha\alpha'} |A^{(j)}(\alpha\alpha' q)|^2 \{(m_q^{(j)} + 1) f_{\alpha'} (1 - f_\alpha)$$
$$- m_q^{(j)} f_\alpha (1 - f_{\alpha'})\} \delta(\varepsilon_{\alpha'} - \varepsilon_\alpha - \omega_{qj}) -$$
$$- \frac{2\pi}{\hbar} \sum_{\alpha\alpha' q'} |B^{(j)}(\alpha'\alpha, -q', q)|^2 (f_\alpha m_q^{(j)} - m_{q'}^{(j)} f_{\alpha'} - f_{\alpha'} f_\alpha (m_q^{(j)} - m_{q'}^{(j)})$$
$$+ (f_\alpha - f_{\alpha'}) m_{q'}^{(j)} m_q^{(j)}\} \delta(\varepsilon_\alpha - \varepsilon_{\alpha'} + \omega_{qj} - \omega_{q'j})$$
$$+ \frac{8\pi}{\hbar} \sum_{\alpha\alpha' q'} |C^{(j)}(\alpha\alpha', -q, -q')|^2 \times \{(m_q^{(j)} + m_{q'}^{(j)} + 1) f_\alpha (1 - f_{\alpha'})$$
$$+ (f_\alpha - f_{\alpha'}) m_q^{(j)} m_{q'}^{(j)}\} \delta(\varepsilon_\alpha - \varepsilon_{\alpha'} - \omega_{qj} - \omega_{q'j}) + I_d\{m_q^{(j)}\} \tag{9}$$

where $m_q^{(j)}$ and f_α are the magnon and electron distribution functions; $I_d\{m_q^{(j)}\}$ is the collision integral including all the remaining (except for the electron–magnon) scattering mechanisms.

Let us consider the deviation $m_{q_z j}^1$ from the steady-state distribution of magnons m_q^j which is achieved as a result of the drift of electrons with a distribution function

$$\bar{f}_\alpha = \left\{\exp\beta\left[\varepsilon_\alpha + \frac{1}{2m^*}(p_z - m^* v_D)^2\right] + 1\right\}^{-1} \tag{10}$$

where v_D is the drift velocity of electrons in the direction of the z axis. Setting $m_q^{(j)} = \bar{m}_q^{(j)} + m_{qj}^1$ and $f_\alpha = \bar{f}_\alpha$, we obtain from Eq. (10) the following equation for $m_{q_z j}^1$:

$$\frac{\partial m_{q_z j}^1}{\partial t} = R_1^{(j)}(q_z)\, m_{q_z j}^1 + R_2^{(j)}(q_z)\, m_{q_z j}^1 + I_d\{m_q^{(j)}\} \tag{11}$$

$$R_1^{(j)}(q) = \frac{2\pi}{\hbar} \sum_{\alpha\alpha'} |A^{(j)}(\alpha\alpha' q)|^2 (\bar{f}_{\alpha'} - \bar{f}_\alpha)\, \delta(\varepsilon_{\alpha'} - \varepsilon_\alpha - \omega_{qj}) \tag{11a}$$

$$R_2^{(j)}(q) = -\frac{8\pi}{\hbar} \sum_{\alpha\alpha' q'} |C^{(j)}(\alpha'\alpha, -q, -q')|^2 \{\bar{f}_\alpha (1 - \bar{f}_{\alpha'})$$
$$+ m_{q'}^{(j)} (\bar{f}_\alpha - \bar{f}_{\alpha'})\} \times \delta(\varepsilon_\alpha - \varepsilon_{\alpha'} - \omega_{qj} - \omega_{q'j}). \tag{11b}$$

In the long-wavelength approximation $\omega_{qj} = \omega_{0j}$, we have assumed in the derivation of Eqs. (11) and (11a) that the contribution of the term $\sim|B|^2$ on the right-hand side of Eq. (9) to the collision integral vanishes since simultaneous creation and annihilation of a magnon with an energy ω_{0j} does not change the steady-state distribution. The quantity $R^{(j)}(q) = R_1^{(j)} + R_2^{(j)}$ introduced in Eq. (11) governs amplification of spin waves from the j-th branch. For an arbitrary strength of the external magnetic field, the matrix elements $\langle\alpha|e^{iqr}|\alpha'\rangle$ in Eqs. (5) and (11a) are given by

$$\langle\alpha|e^{iqr}|\alpha'\rangle = \delta_{p_y^{\alpha'}, p_y^\alpha + q}\, \delta_{p_z^{\alpha'}, p_z^\alpha + q_z} (n!\, n'!)^{-1/2}$$
$$\times \exp(-q_\perp^2 \rho_0^2/4)(q_\perp^2 \rho_0^2/2)^{\frac{|n'-n|}{2}} L_n^{|n'-n|}(q_\perp^2 \rho_0^2/2). \tag{12}$$

where $q_\perp^2 = q_x^2 + q_y^2$; $L_n^{|n'-n|}$ are generalized Laguerre polynomials; $\rho_0 = \sqrt{2\mu_B c\hbar/eH}$ is the magnetic length.

3. AMPLIFICATION OF SPIN WAVES IN THE ULTRAQUANTUM LIMIT

We shall consider the conditions under which the low-frequency branch (j = 2) is amplified. In the quantum limit when the cyclotron frequency satisfies $\hbar\Omega \gg \hbar\omega_0$, we can restrict ourselves to the first Landau level in the expressions (11) and (12). Setting then n = n' = 0, we obtain from Eqs. (5) and (12)

$$|A^{(2)}(\alpha\alpha' q_z)|^2 = Q^2 \delta_{q_z, p_z' - p_z} \delta_{p_x', p_x}. \tag{13}$$

$$|C^{(2)}(\alpha\alpha', -q', -q_z)|^2 = Z^2 e^{-q_\perp'^2 \rho_0^2/2} \delta_{p_z' - p_z, -q_z - q_z'} \delta_{p_x' - p_x, -q_x'}, \tag{14}$$

where $Q_q = Q$ and $Z_{qq'}^{(2)} = Z$ is independent of q and q'. Substituting Eq. (13) in Eq. (11a) and using the fact that the summation over p_x gives rise to a factor $L_x L_y m^* \Omega / 2\pi\hbar$, we obtain the following expression for $R_1(q_z)$:

$$R_1(q_z) = \frac{Q^2 V m^{*2} \Omega}{\hbar^4 2\pi |q_z|} \Delta \tag{15}$$

$$\Delta = f^0\left(p_z - \frac{m^* \omega_{02}}{q_z} - p_D + \frac{\hbar q_z}{2}\right) - f^0\left(p_z = \frac{m^* \omega_{02}}{q_z} - p_D - \frac{\hbar q_z}{2}\right) \tag{16}$$

where f^0 is the equilibrium electron distribution function and V is the volume of the system. When the wave vector satisfies the inequality $\hbar^2 q_z^2 / 8m^* T \gg 1$, we can approximate the expression (16) for a nondegenerate electron gas by

$$\Delta \approx e^{(\mu - \hbar\Omega/2)/T} \frac{\hbar \omega_{02}}{T} \left(1 - \frac{q_z v_D}{\omega_{02}}\right) \exp\left[-\frac{\omega_{02}^2 m^*}{2 q_z^2 T} \left(1 - \frac{q_z v_D}{\omega_{02}}\right)^2\right] \tag{17}$$

where μ is the chemical potential.

When considering R_2, we note that the expression (11b) for R_2 contains the steady-state magnon distribution function which is, in general, not known because of the term $I_d\{m_q\}$ and can be only estimated. Assuming that $\tilde{m}_q$ under electron drift conditions differs only insignificantly from the equilibrium value, i.e., setting $\tilde{m}_q \approx m_q^0$ and $m^* v_D^2/2 \ll T$, we can use the expression (14) and obtain from Eq. (11b) the following expression for R_2:

$$\left.\begin{aligned} R_2(q_z) &= -\frac{\Omega}{\pi^3} \frac{m^{*2} V^2 Z^2}{\rho_0^2 \hbar^4} e^{(\mu - \hbar\Omega/2)} K_0(\hbar\omega_{02}/T) \\ K_0(x) &\sim \begin{cases} \sqrt{\pi \cdot 2x}\, e^{-x}, & x \gg 1 \\ -\ln 2x, & x \ll 1 \end{cases} \end{aligned}\right\} \tag{18}$$

It follows from Eqs. (15)-(18) that $R_1(q_z) < 0$ for $\omega_{02} - q_z \cdot v_D > 0$ and $R_1(q_z) > 0$ for $\omega_{02} - q_z v_D < 0$, whereas $R_2(q_z)$ is always negative. In particular, it follows that high-frequency branch is always damped under the aforementioned conditions. Hence, the necessary condition for amplification of spin waves for $\omega_{02} - q_z v_D < 0$ is that

$$\Lambda = |R_1(q_z)| / |R_2(q_z)| \gg 1 \tag{19}$$

In the case $H^2/H_E^2 \ll 1$, which is of special interest in practice (typical values of H_E for antiferromagnets are 10^6-10^7 Oe) when $q_z v_D \gg \omega_{02}$, we can use Eqs. (15)-(18) and the expressions (6) for Q and Z to obtain the following expression for Λ:

$$\Lambda \approx \frac{4\pi^2 S}{K_0(\omega_{02}/T)} \frac{\hbar v_D}{aT} \left(\frac{\rho_0}{a}\right)^2 \frac{H_\parallel^2}{H^2} \tag{20}$$

where a is the lattice constant. The magnetic field strength in Eq. (19) is bounded from below by the requirement that $H \gg H_\parallel$ ($\Omega \gg \omega_0$). For example, if $T = 20$ K, $H_\parallel = 5 \cdot 10^4$ Oe, $H = 5 \cdot 10^5$ Oe, $v \approx 10^6$ cm/sec, $a = 3 \cdot 10^{-8}$ cm, $\omega_0 = 10^{-3}$ eV, $m^* = m_0$, and $S = 2$, we find that $\Lambda \sim 20$, i.e., the amplification criterion (19) is well satisfied. The following condition represents a more stringent restriction on the amplification of spin waves in practice:

$$\eta > \gamma \tag{21}$$

where $\eta = (R_1 + R_2)/\omega_{02}$ is the gain of spin waves; γ is the damping decrement determined by the collision integral $I_d\{m_q\} \sim \gamma m_q^{-1}$. Using the same values of the parameters as in the estimate of Λ and choosing $H_E = 10^6$ Oe and $A = 0.5$ eV, we find that for carrier densities $n \sim 5 \cdot 10^{17}$ the value of η amounts to $\eta \approx R_1/\omega_{02} \sim 10^{-1}$, i.e., it is an order of magnitude greater than the damping decrement γ obtained from estimates of damping of spin waves in yttrium ferrites which is $\gamma \sim 10^{-2}$-10^{-3} (see Ref. 1).

4. DISCUSSION OF RESULTS

Our results solve the problem of finding the necessary conditions for the amplification of spin and electromagnetic waves in uniaxial antiferromagnets. A current of electrons generated by a strong electric field parallel to the anisotropy axis of an antiferromagnet leads to the creation of magnons belonging to the second resonance branch with a frequency $\omega = \omega_2$ which then interact with an electromagnetic wave whose magnetic component is parallel to the magnetic field. An electromagnetic wave whose magnetic component is perpendicular to the magnetic field interacts with the first resonance branch, which leads to the usual absorption due to an antiferromagnetic resonance. The same conditions are obtained for the amplification in all other cases when the antiferromagnetic vector is perpendicular to the magnetic field, i.e., when the amplification occurs only for waves with a longitudinal magnetic component.

No reliable experimental data are at present available to indicate that the amplification of spin waves occurs in magnetically ordered crystals. This is clearly due to the fact that all experiments were carried out exclusively for ferromagnets when the relativistic amplification mechanism predominates. This mechanism cannot satisfy the amplification criterion (21). A strong s–f exchange interaction of carriers with the magnetization in ferromagnets then yields only an insignificant contribution to the total amplification compared with the relativistic amplification and tends to zero exponentially when the interaction increases for $AS \gg T$, i.e., $R_F \propto \exp(-AS/T)$.

Comparing the gain for antiferromagnets describdd by Eqs. (15) and (18) with the available estimates of R_F for ferromagnets (due to the relativistic interaction) at the same drift velocities, we obtain the following ratio:[1)] $R_{AF}/R_F \sim 10^4$-10^5. It is also important to note that, in many antiferromagnets, the frequency range corresponding to an antiferromagnetic resonance is located in the far infrared region whereas a resonance in ferromagnets can occur only in the microwave or the already mastered radiowave ranges.

The author is grateful to V. G. Veselago and K. M. Golant, who have drawn his attention to the importance to the amplification of spin waves in magnetically ordered crystals.

[1)] We note that the collisionless limit considered ($ql \gg 1$, where l is the mean free path of electrons) assumes antiferromagnets with high carrier mobilities (for example, $Hg_{1-x}Mn_xTe$ with $u \sim 10^5$ cm$^2 \cdot$V$^{-1} \cdot$sec^{-1}). The opposite limiting case $ql \ll 1$ is usually realized in antiferromagnets with low carrier mobilities ($u \lessdot 10$-10^2 cm$^2 \cdot$V$^{-1} \cdot$sec^{-1}). This case lies outside the scope of the present investigation and needs to be studied separately. Nevertheless, it is possible that the gain in antiferromagnets exceeds, even in this case, the corresponding gain for ferromagnets by several orders of magnitude.

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Translated by D. Mathon